Understanding the complexity of frequency and phase angle fluctuations in power grids

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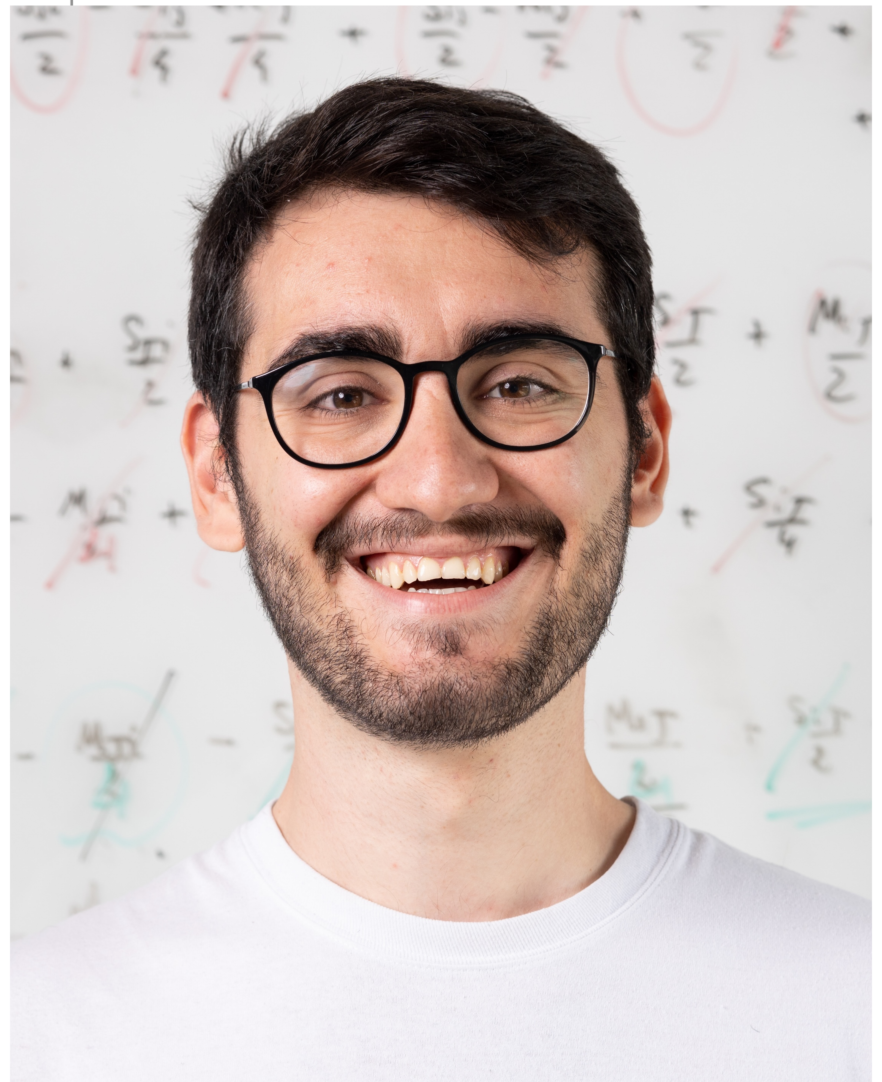
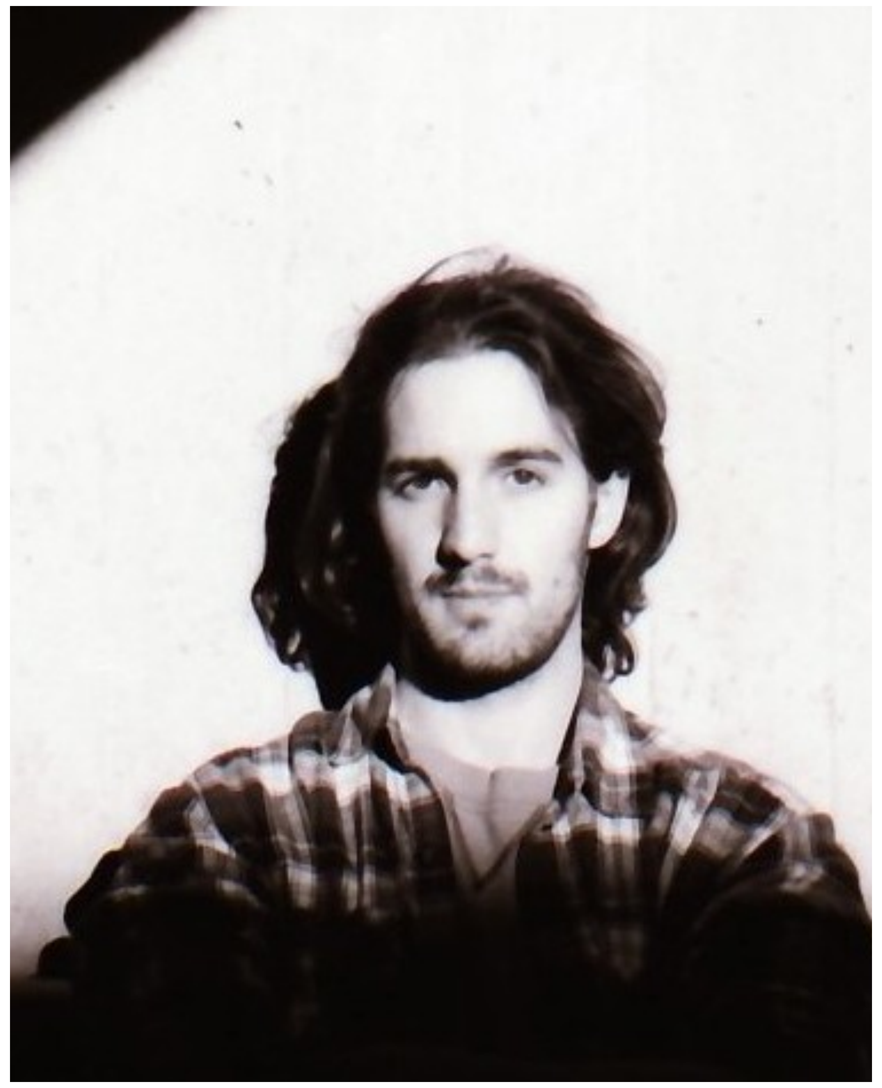
Alessandro Lonardi,[1] Jacques M. Maritz,[2] Leonardo Rydin Gorjão,[3,4] and Christian Beck[1]

Preprint	arXiv:2604.03133
Code (GitHub)	aleable/power-grid-complexity
Data (Zenodo)	10.5281/zenodo.19397526
Slides	aleable.github.io/talks.html





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Stability of the South African Power Grid: A data-driven statistical physics approach

UK Research and Innovation


Christian Beck
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Chantelle Van Staden
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Cristina Trois
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Jacques Maritz
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Understanding the complexity of frequency and phase angle fluctuations in power grids

Keywords

- Superstatistics
- Power grid, swing equations

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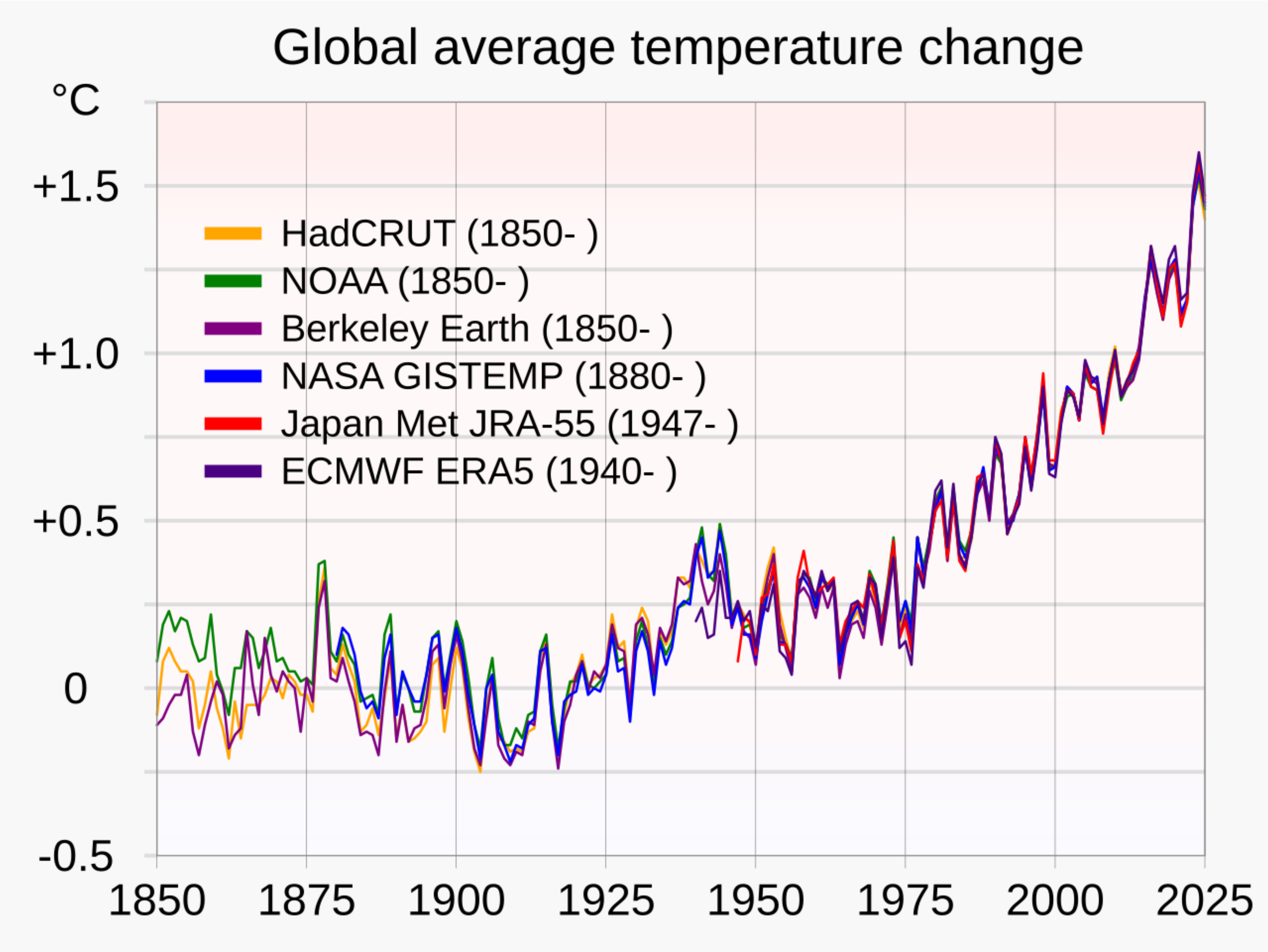
Broad scope of the talk

- Show that statistical physics is a helpful tool for describing the emergent properties of power grids

Why does it matter?

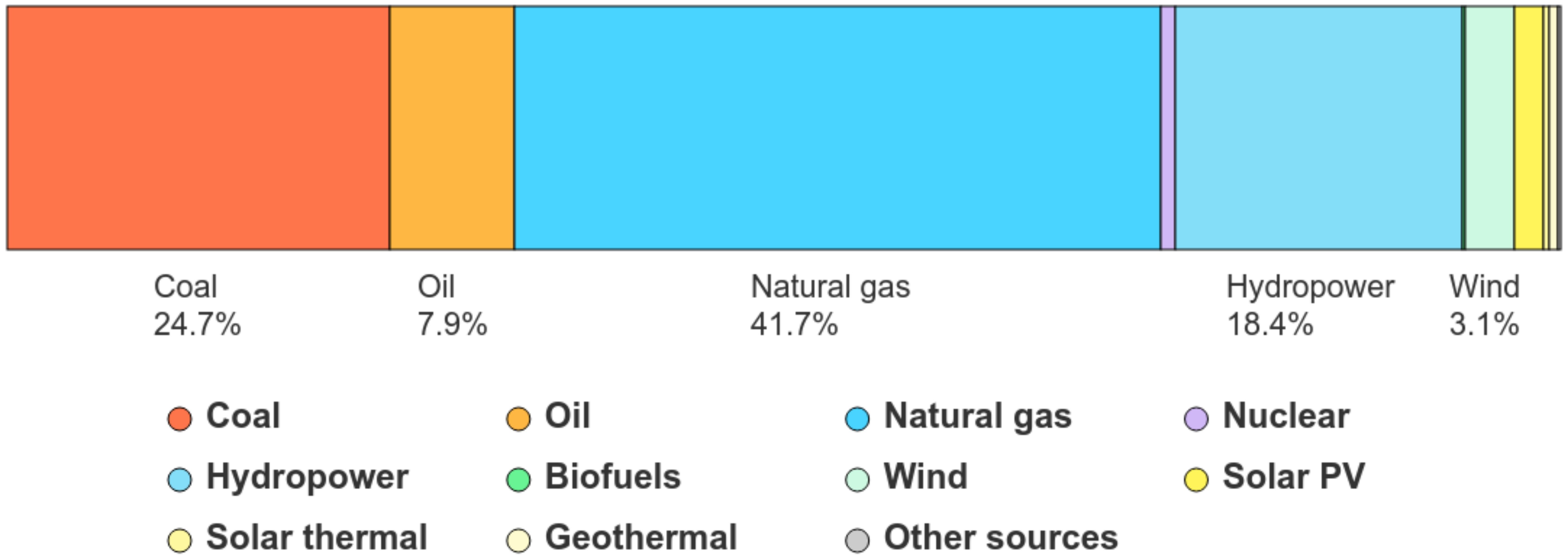
Increasing grid investment [helps] [...] holding the increase in the global average temperature to well below 2°C above pre-industrial levels and pursuing efforts to limit the temperature increase to 1.5°C.

Adapted, United Nations Climate Change Conference, COP29 Global Energy Storage and Grids Pledge (2024)



https://climate.metoffice.cloud/temperature.html

Electricity generation by source, Africa, 2023



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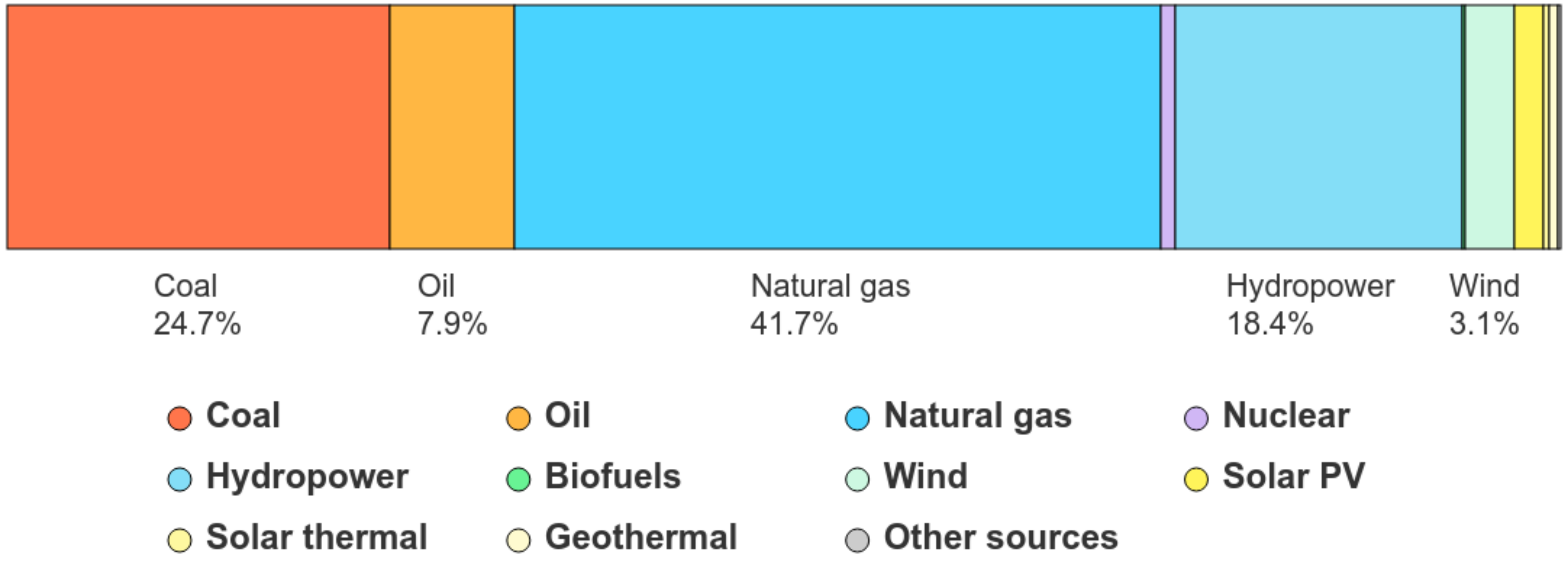
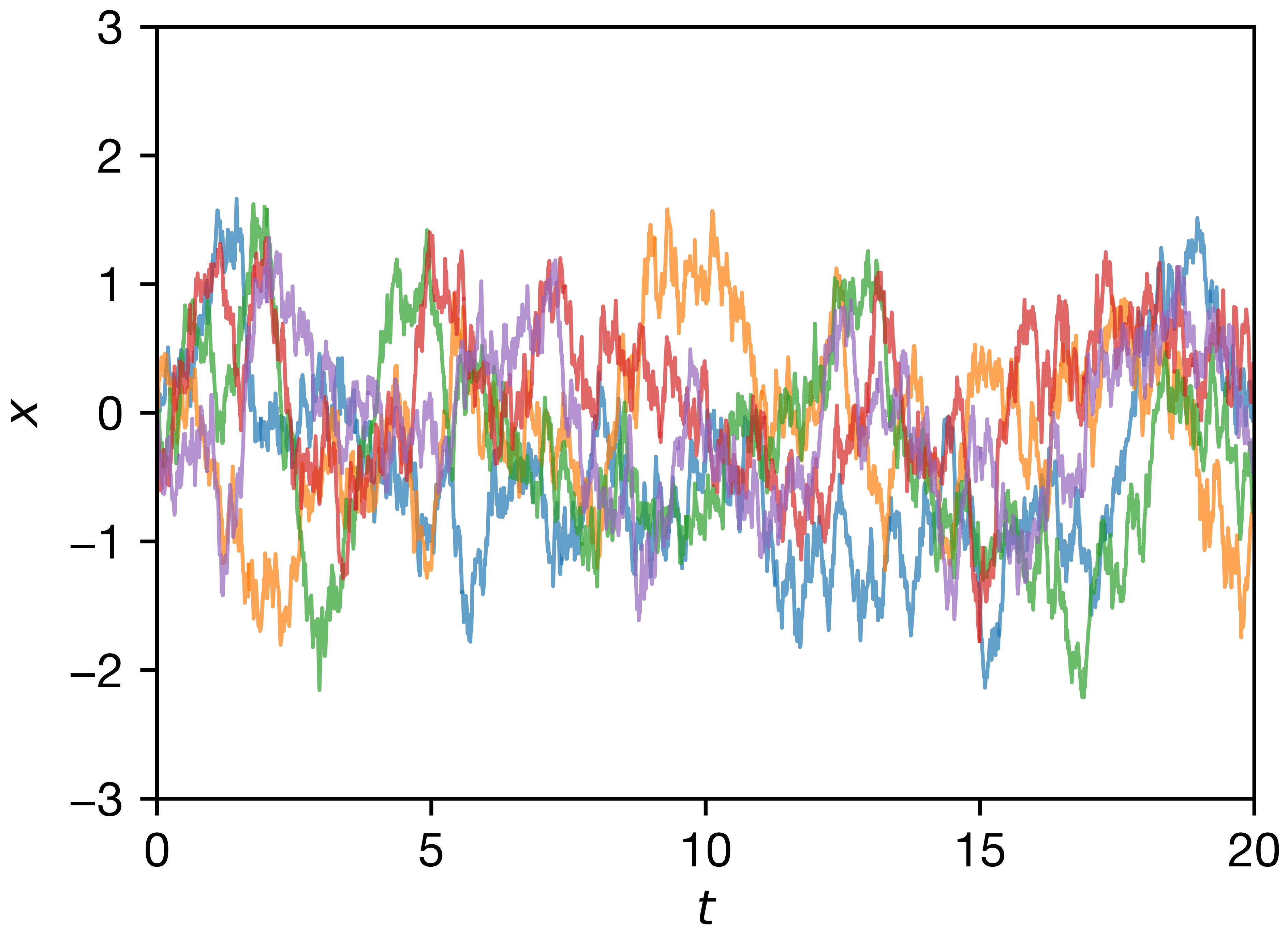


Statistical physics

Langevin equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\gamma x + \epsilon\xi$$

$$\gamma > 0$$
$$\epsilon > 0, \xi \sim \mathcal{N}(0,1)$$
$$\beta := \gamma/\epsilon^2$$



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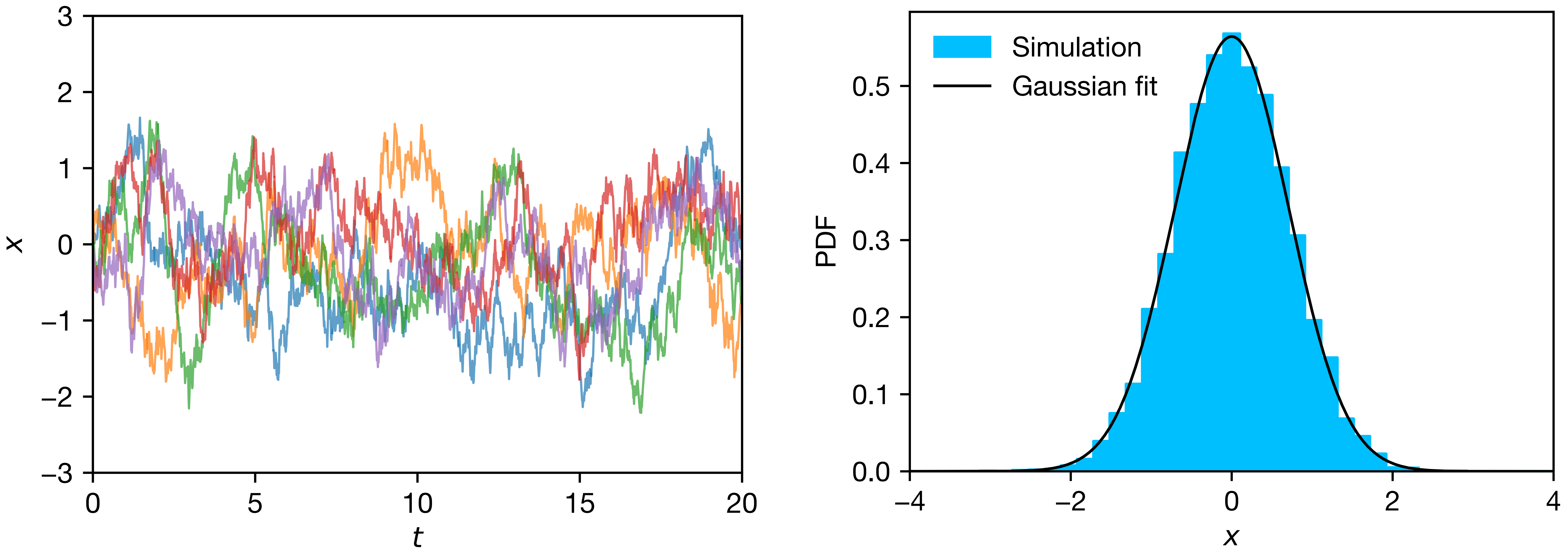
$$p(x) \propto e^{-\beta x^2}$$



Superstatistics: what happens if β varies slowly?

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\gamma x + \epsilon\xi$$

$\beta(t)$ slow

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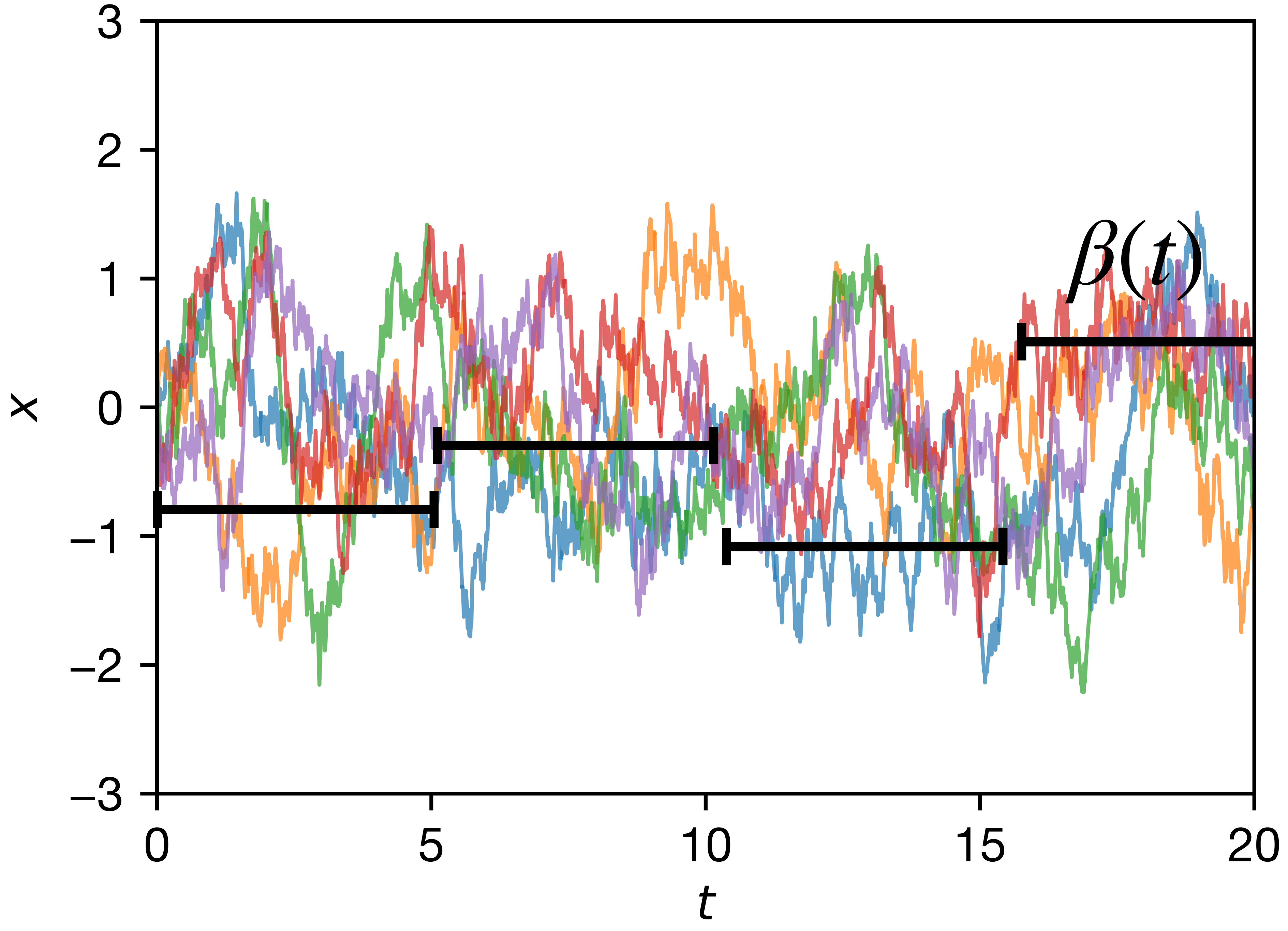

C. Beck *Physical Review Letters* 2001
C. Beck, E. G. D. Cohen *Physica A: Statistical Mechanics and its Applications* 2003

Superstatistics: what happens if β varies slowly?

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\gamma x + \epsilon\xi$$

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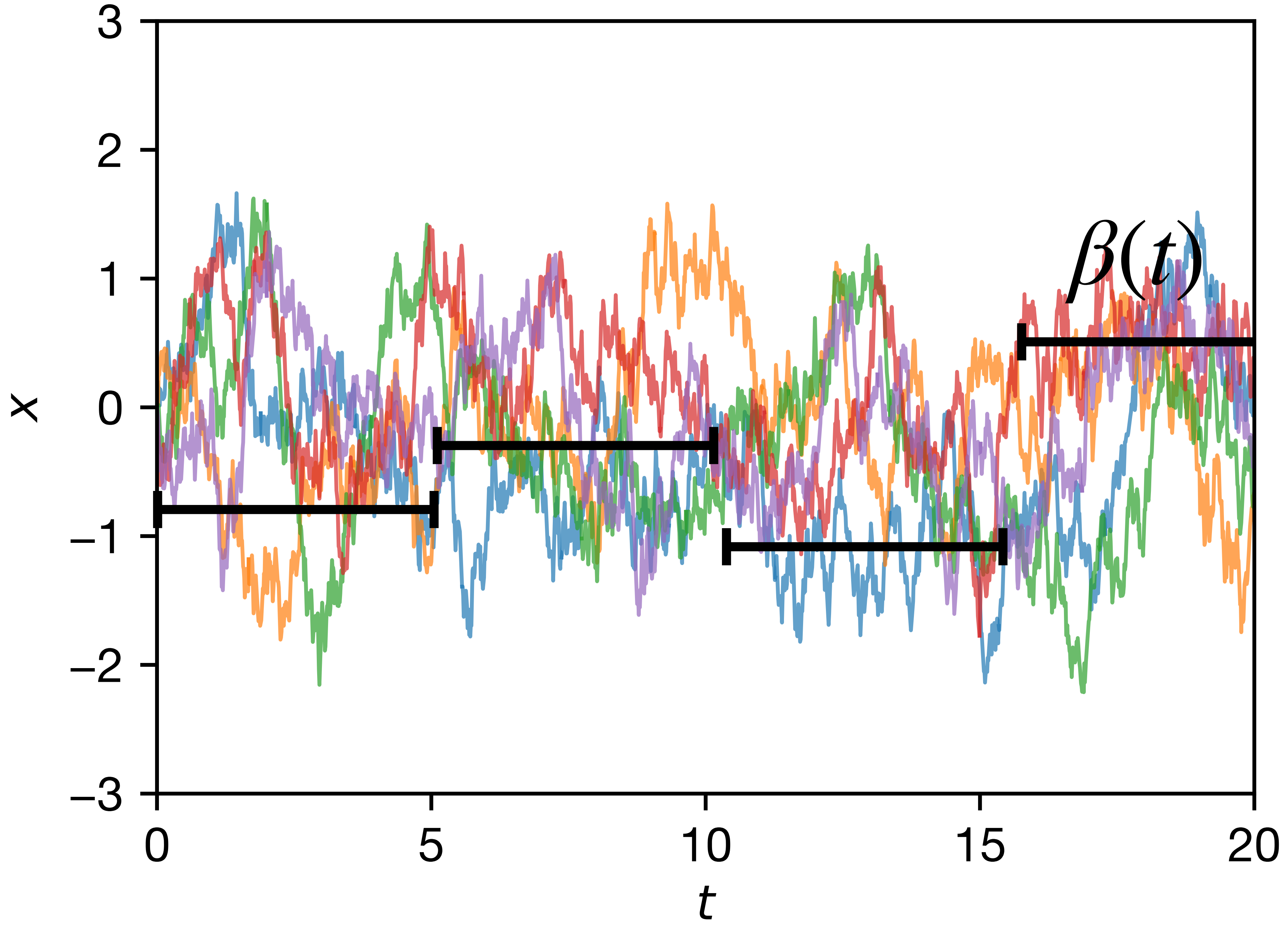
$$p(x) \propto \int f(x \mid \beta)\, \varphi(\beta)\, \mathrm{d}\beta$$



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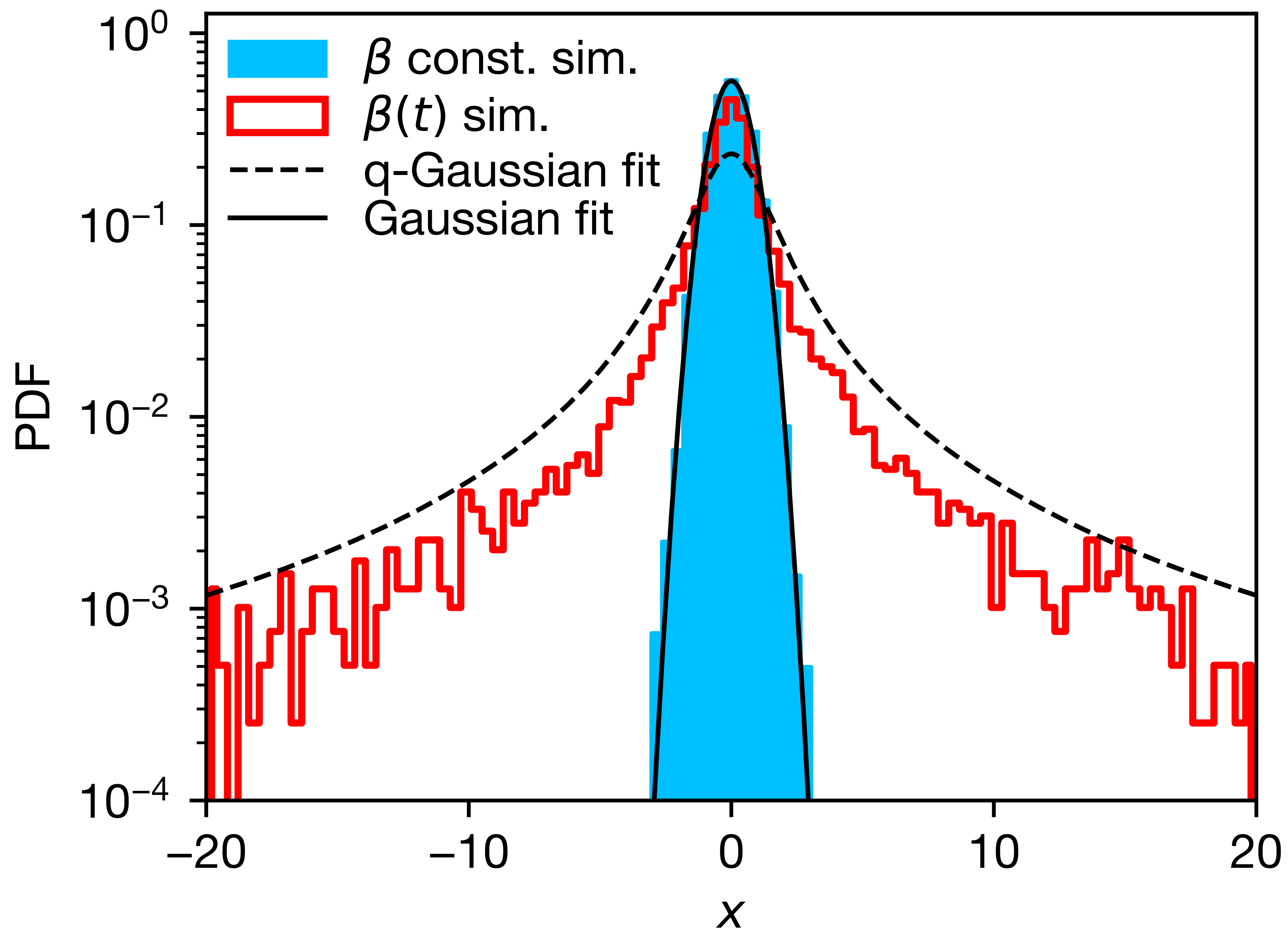

$\varphi(\beta) = \chi^2$

$f(x \mid \beta)$ = Gaussian

$p(x) = q$-Gaussian

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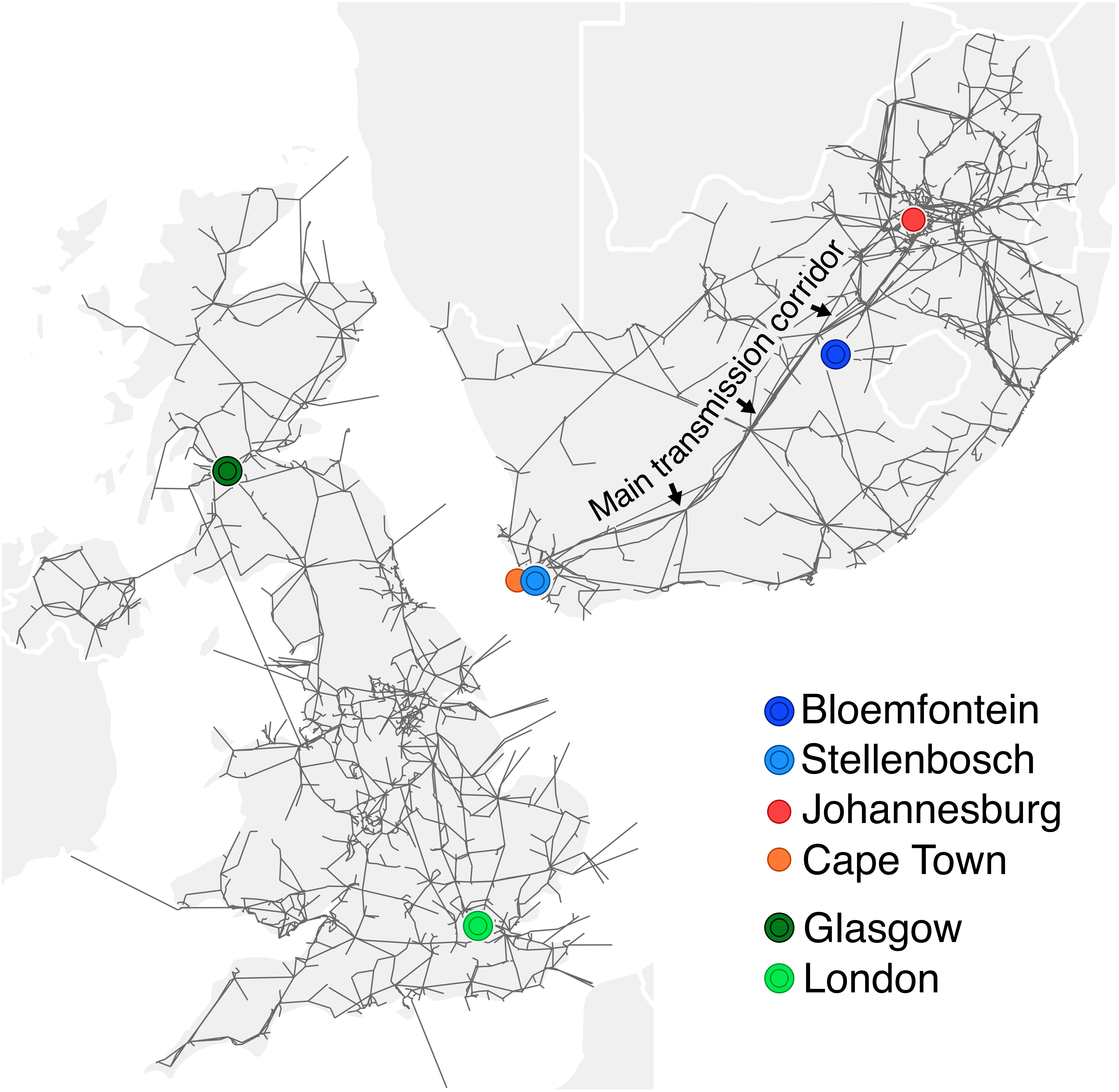
Power grids

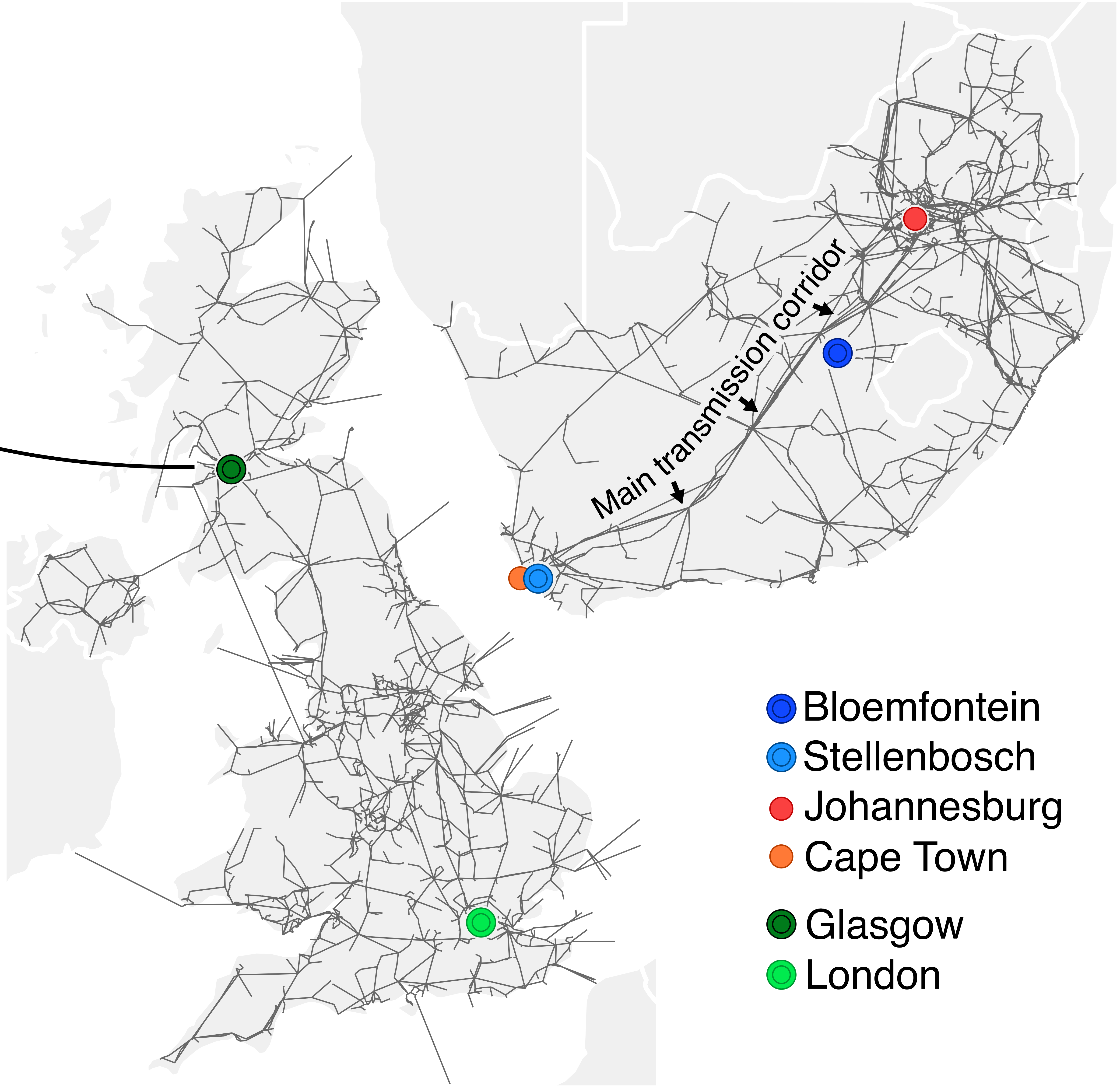
Power grids (as a complex system)



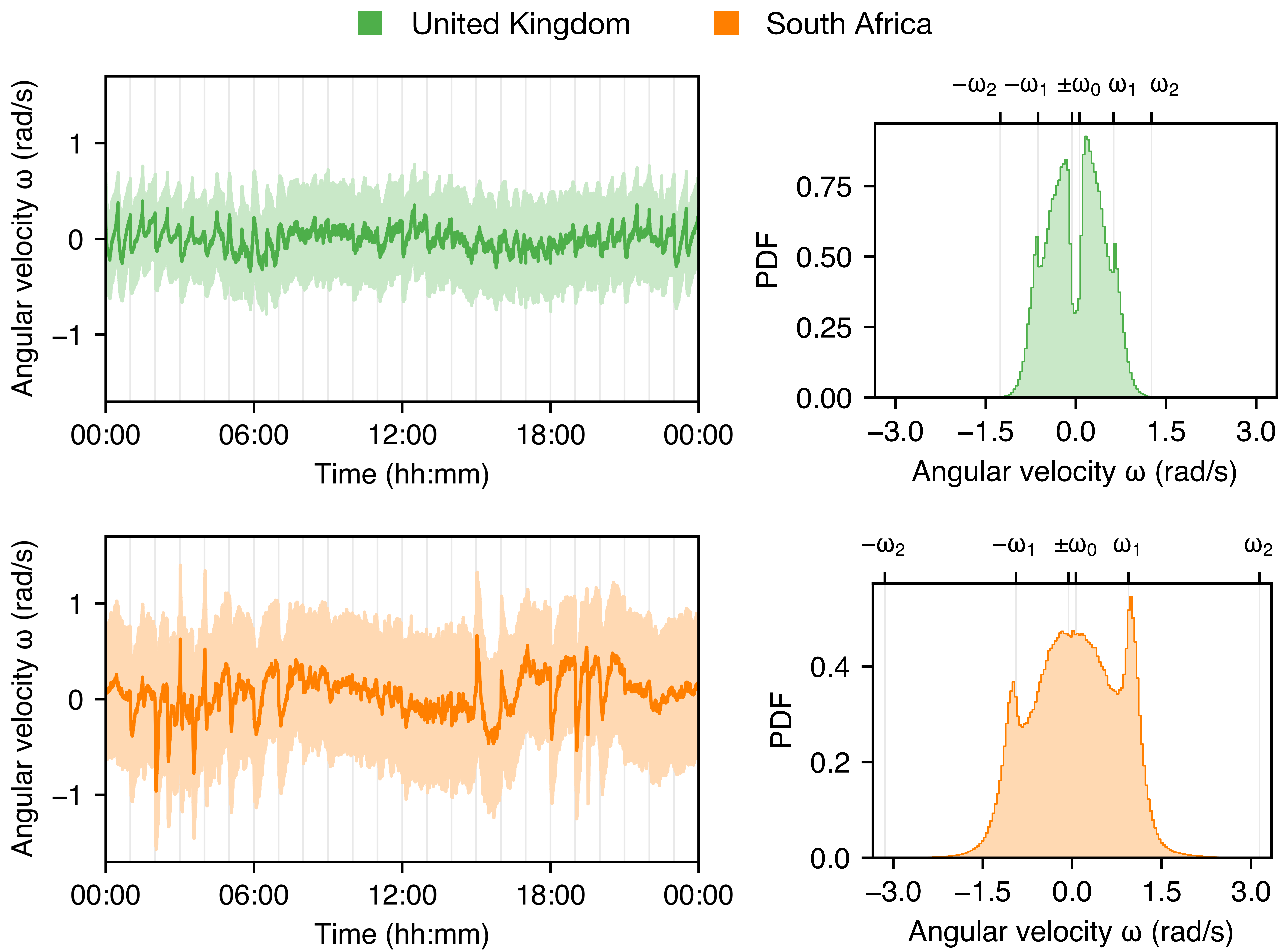
Power grids (as a complex system)

$$\omega(t) = 2\pi\,(f(t) - f_{\mathrm{R}})$$

$$\theta_i(t) \in [0,2\pi)$$

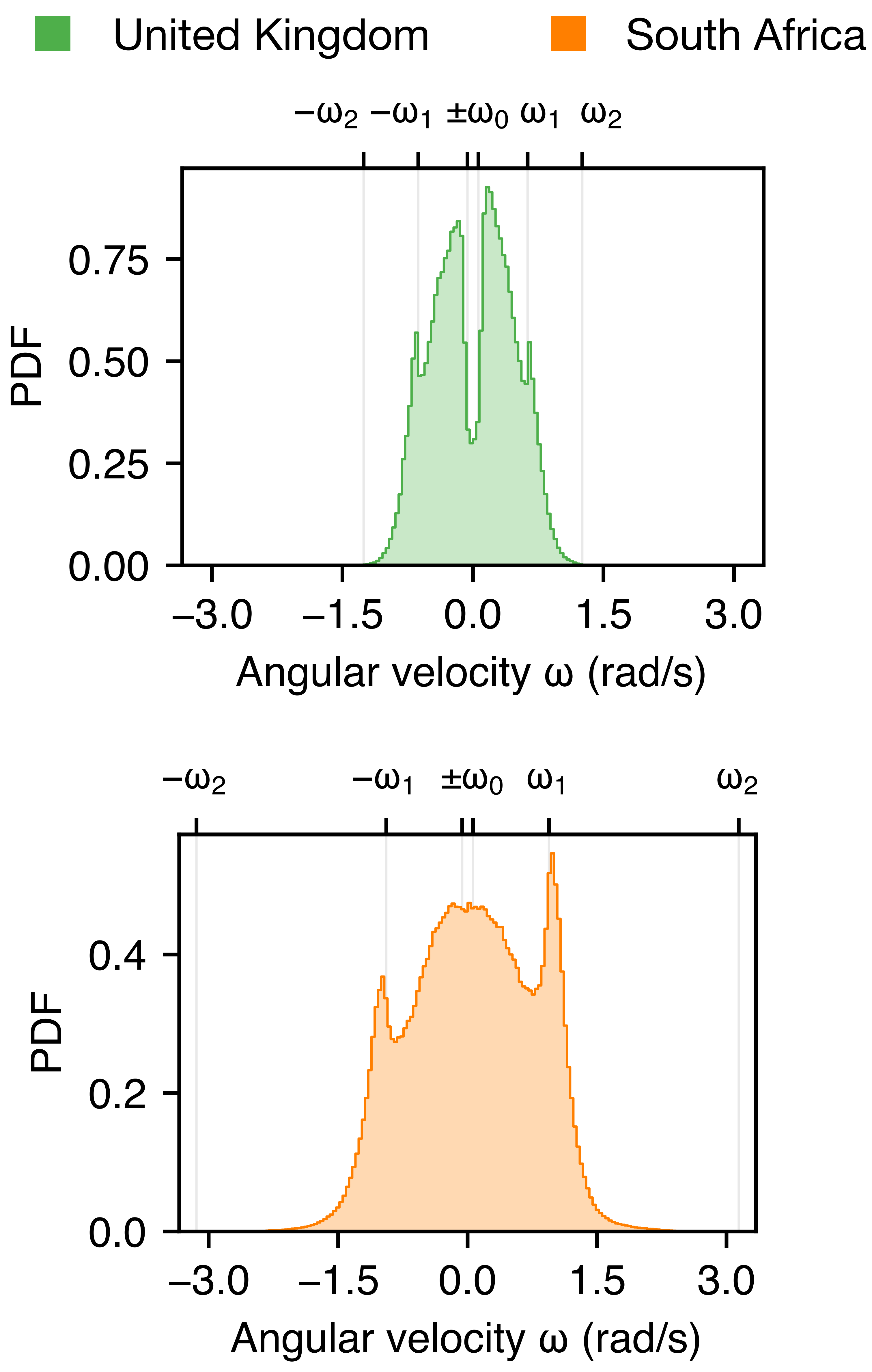


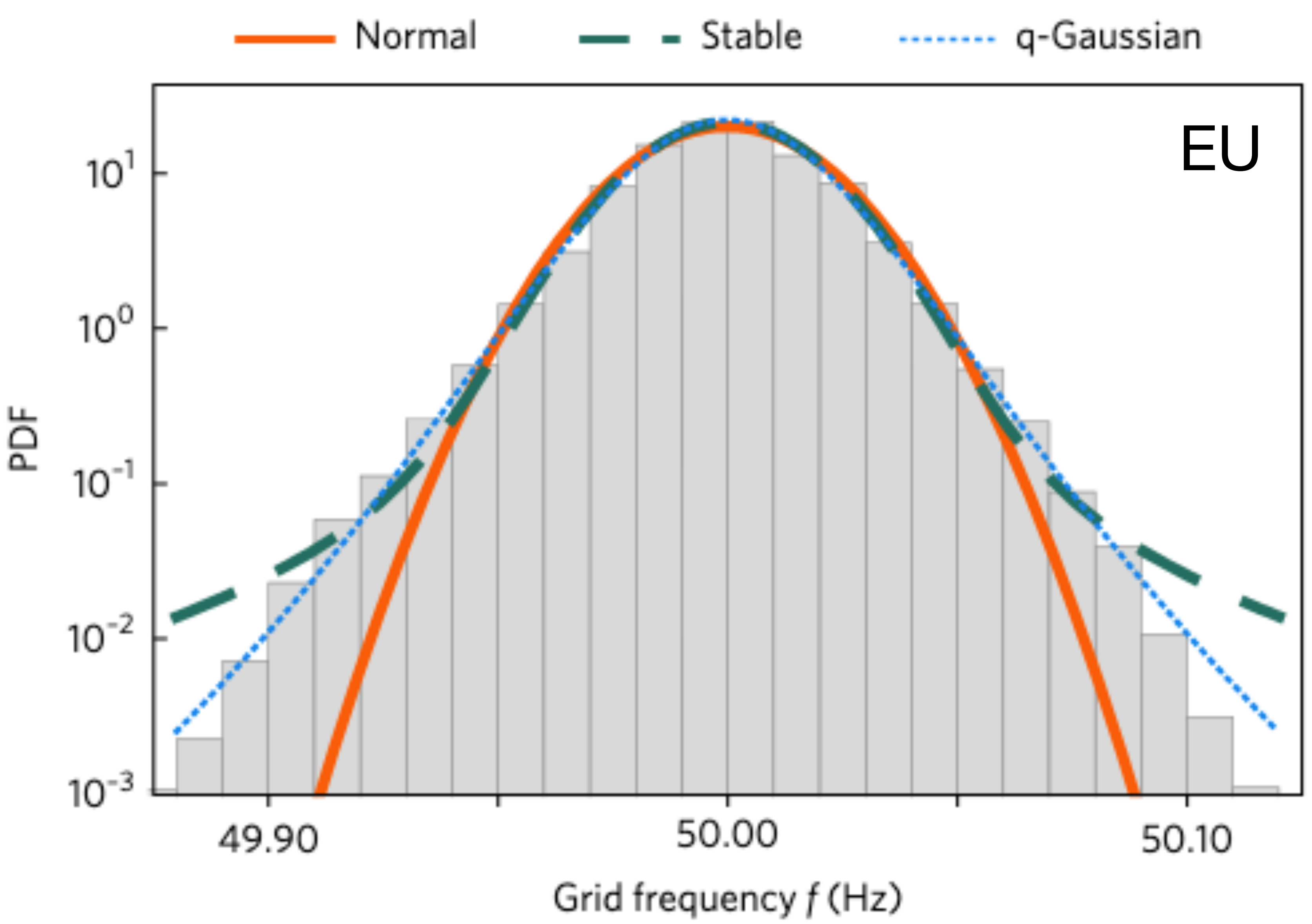
Frequency



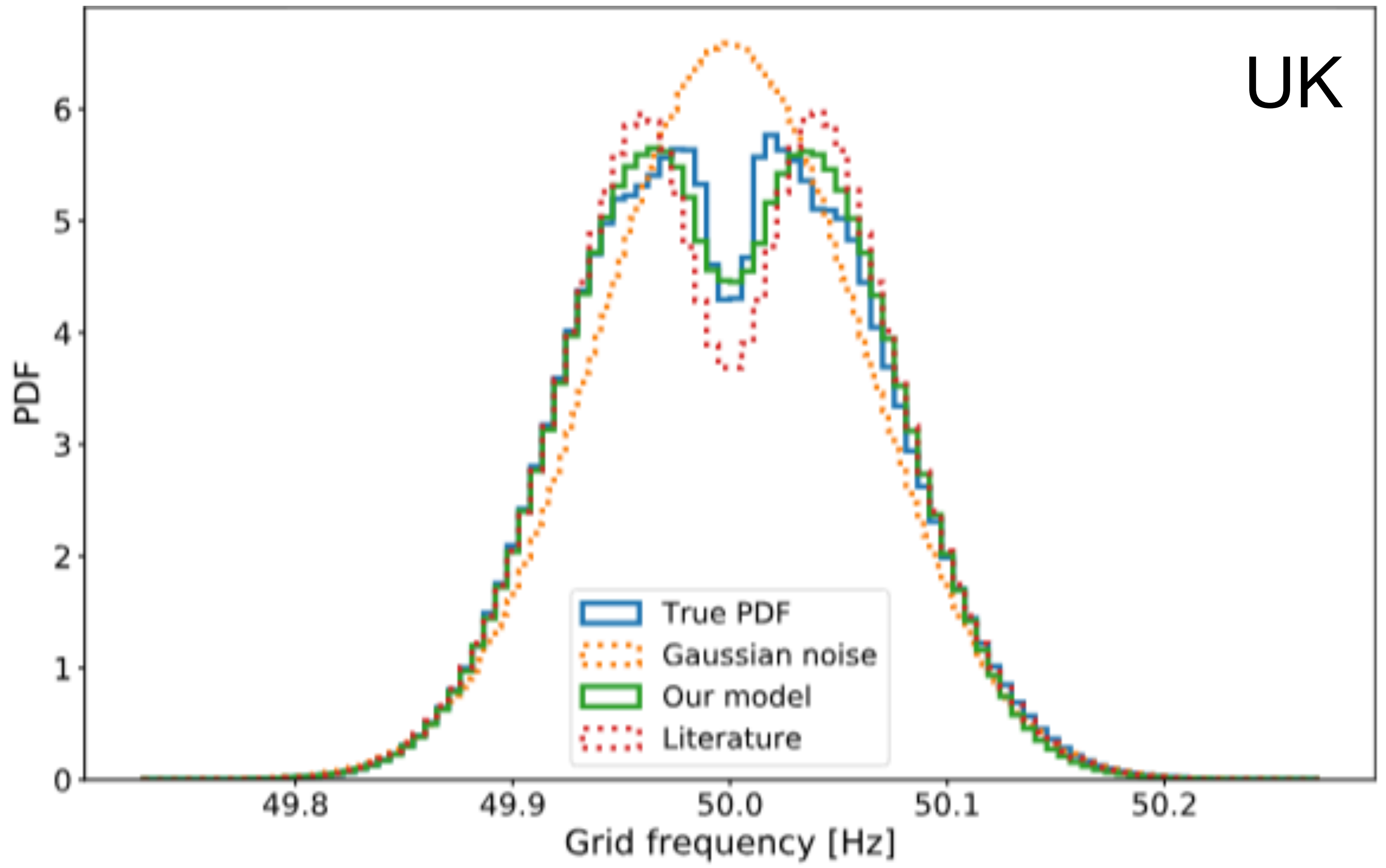
United Kingdom
South Africa
Angular velocity ω (rad/s)
Time (hh:mm)
00:00
06:00
12:00
18:00
00:00
PDF
−ω2 −ω1 ±ω0 ω1 ω2
Angular velocity ω (rad/s)

New data insights departing from previous findings





B. Schäfer et al. *Nature Energy* (2018)



D. Kraljic *IEEE Transactions on Power Systems* (2023)

We need to piece the puzzle together…

A new kind of superstatistics

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = H(\omega) + P(t) + \epsilon\xi$$

$$\frac{dx}{dt} = -\gamma x + \epsilon\xi$$

$\beta := \gamma/\epsilon^2 \quad \beta(t)$ slow

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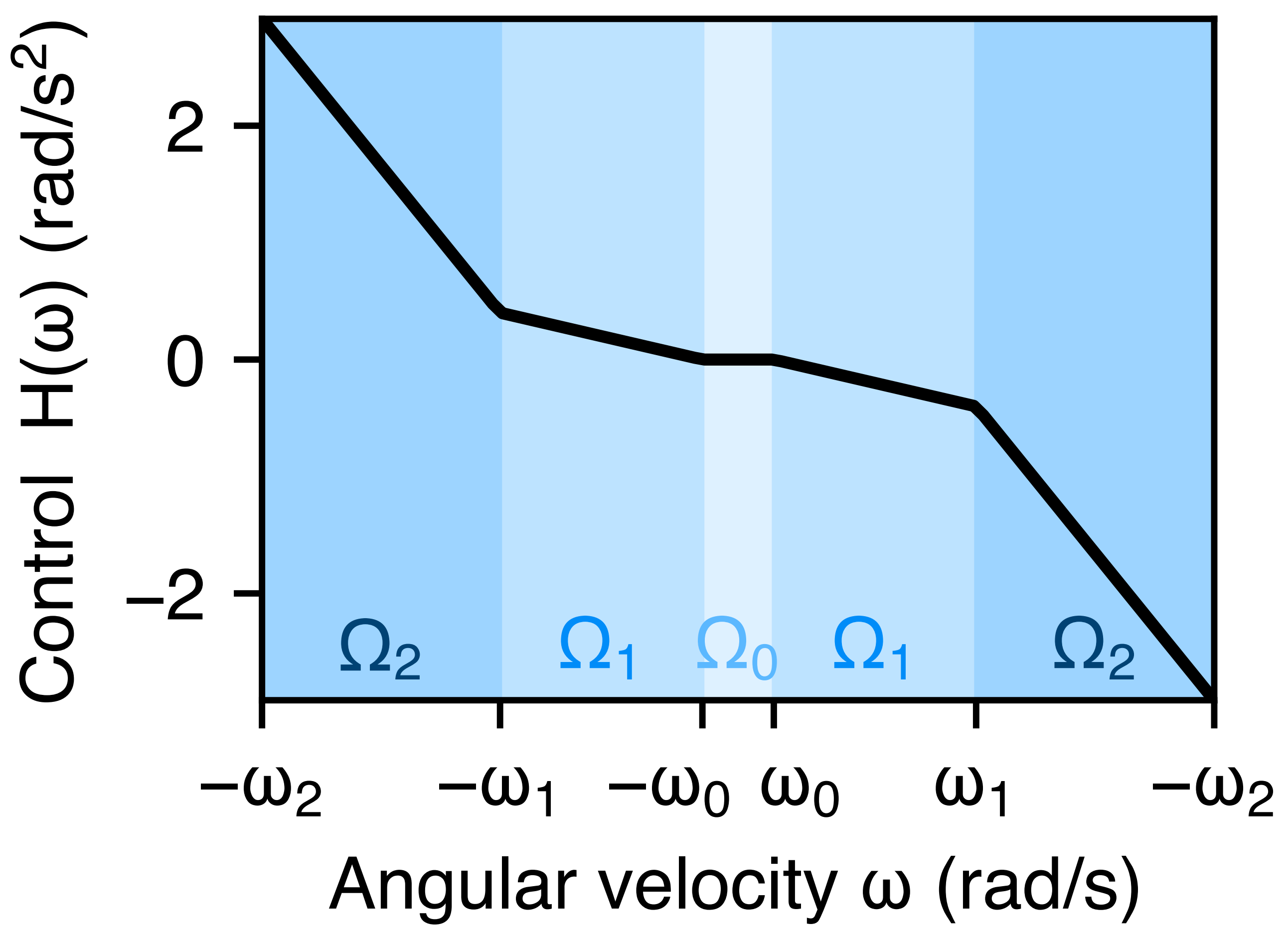
Nonlinear control



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$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = H(\omega) + P(t) + \epsilon\xi$$

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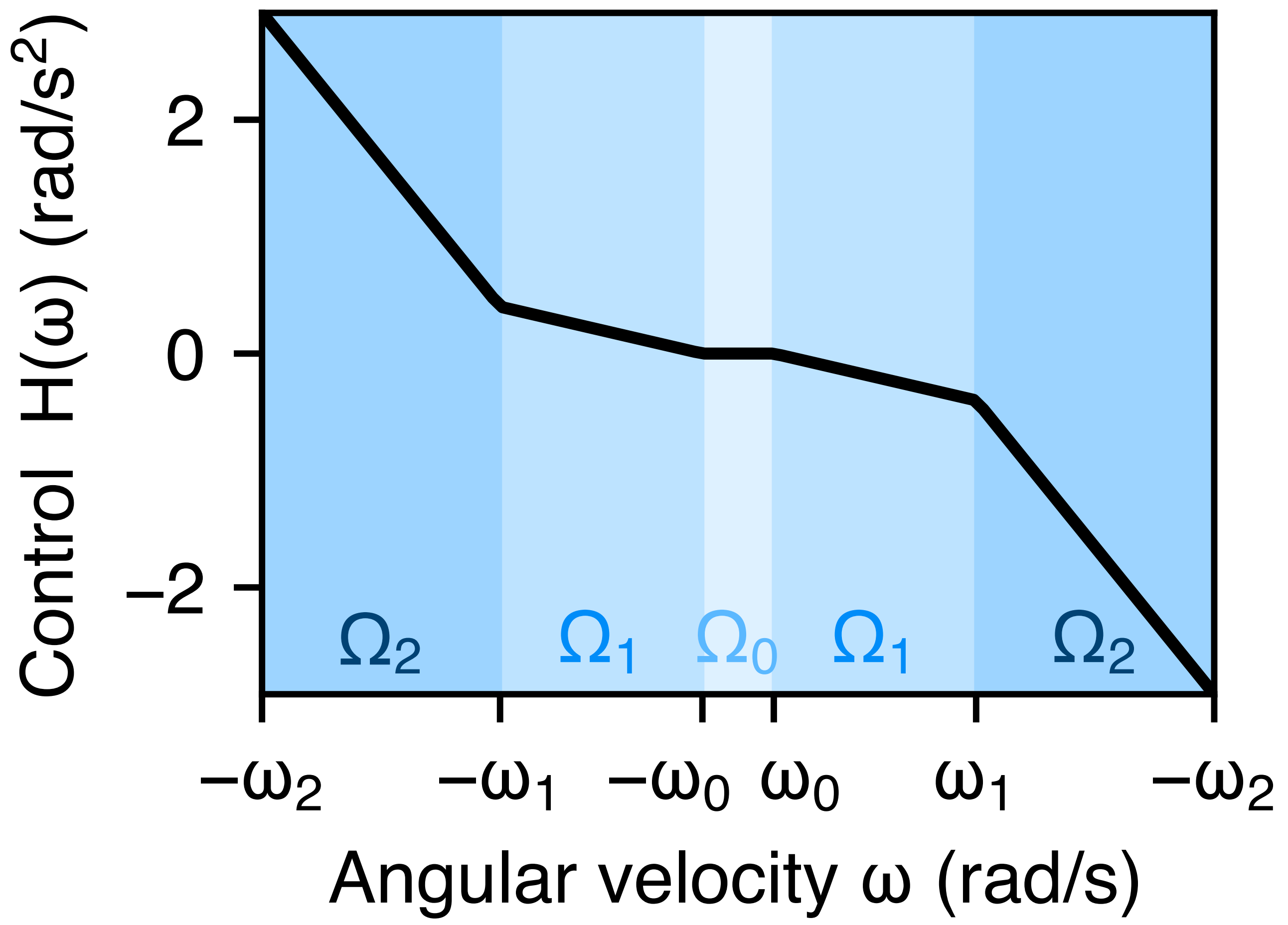
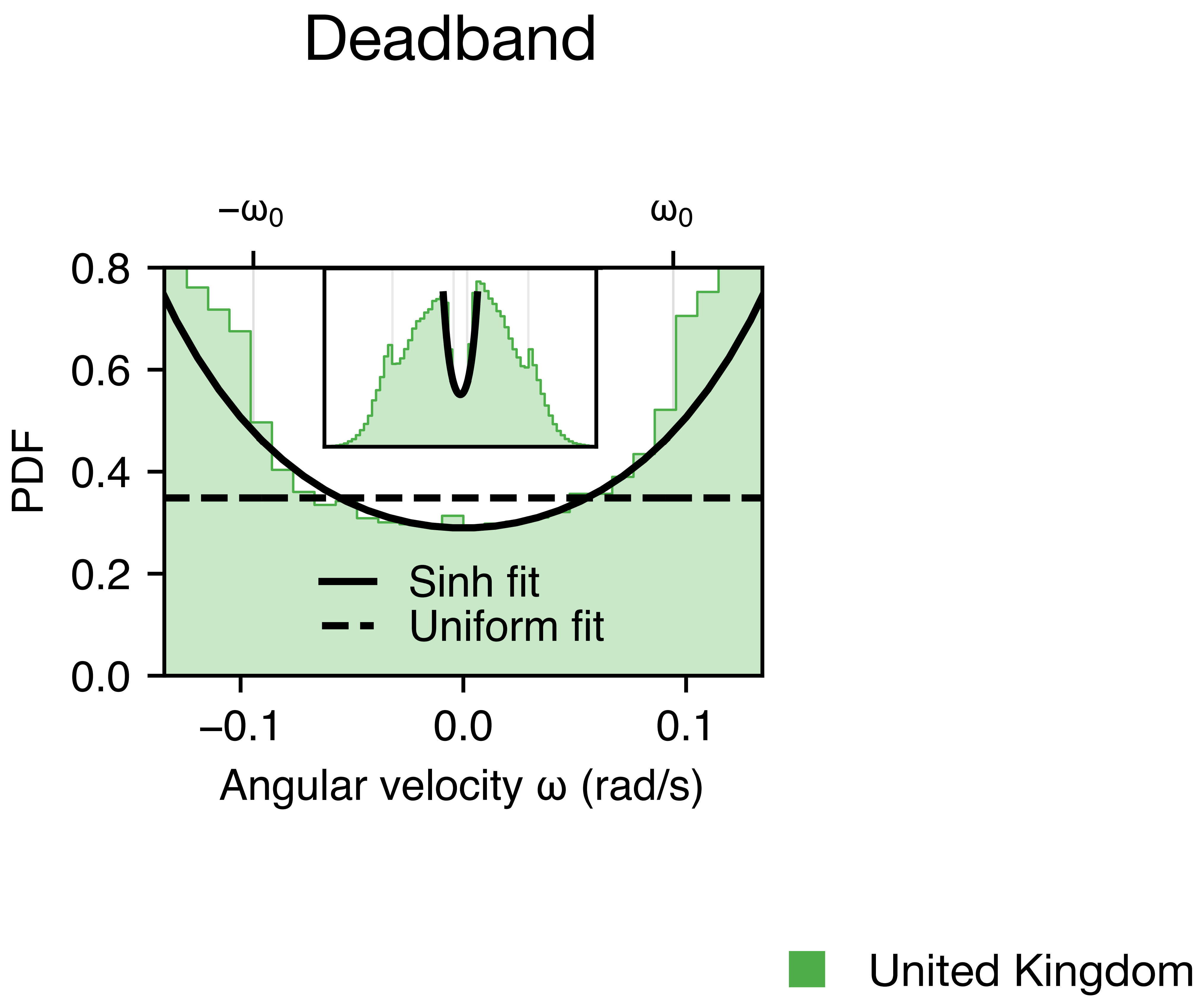
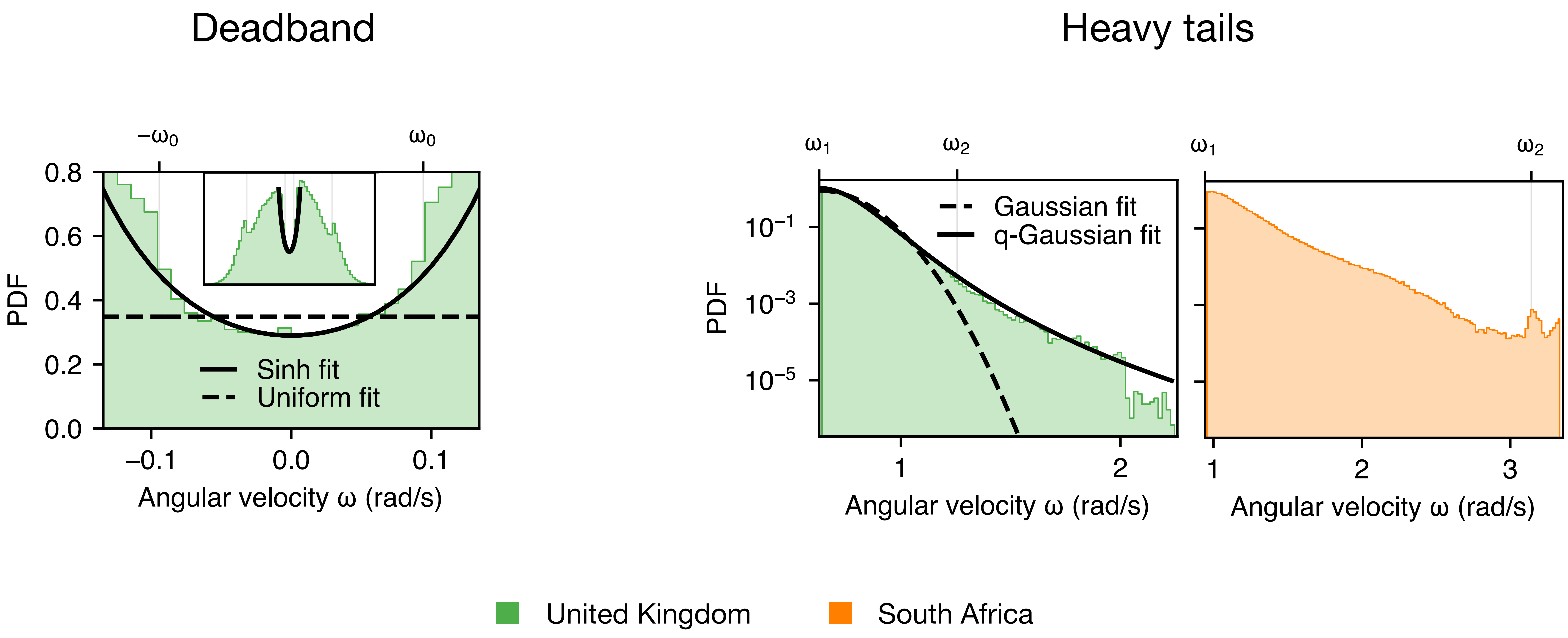
$\epsilon > 0, \xi \sim \mathcal{N}(0,1)$

$P(t)$ is the slow “superstatistical” variable → $p(\omega) \propto \int f(\omega \mid P)\varphi(P)\,\mathrm{d}P$

Nonlinear control

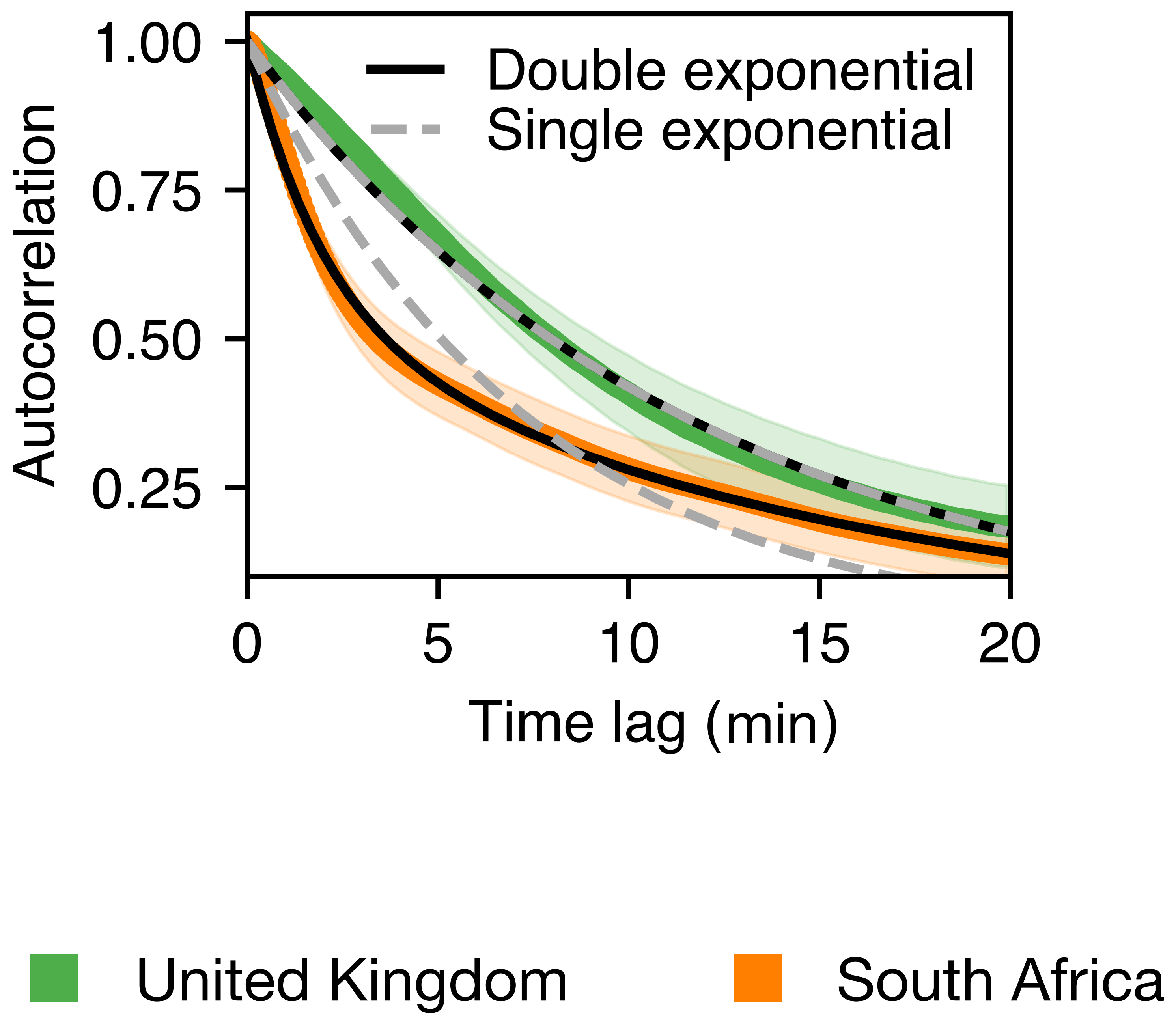



Deadband
−ω0
ω0
0.8
0.6
0.4
0.2
0.0
PDF
Sinh fit
Uniform fit
−0.1
0.0
0.1
Angular velocity ω (rad/s)
United Kingdom



Deadband
Heavy tails
$-\omega_0$
ω_0
0.8
0.6
0.4
0.2
0.0
PDF
Sinh fit
Uniform fit
−0.1
0.0
0.1
Angular velocity ω (rad/s)
ω_1
ω_2
10^{-1}
10^{-3}
10^{-5}
Gaussian fit
q-Gaussian fit
1
2
3
United Kingdom
South Africa

Autocorrelation decay



$$\mathrm{ACF}(\tau) = \frac{\mathbb{E}[(\omega(t) - \overline{\omega})(\omega(t-\tau) - \overline{\omega})]}{\mathrm{Var}(\omega(t))}$$

$$\mathrm{ACF}(\tau) \sim e^{-\gamma\tau}$$

$$\mathrm{ACF}(\tau) \sim A\, e^{-\gamma_1\tau} + (1 - A)\, e^{-\gamma_2\tau}$$

Phase angles

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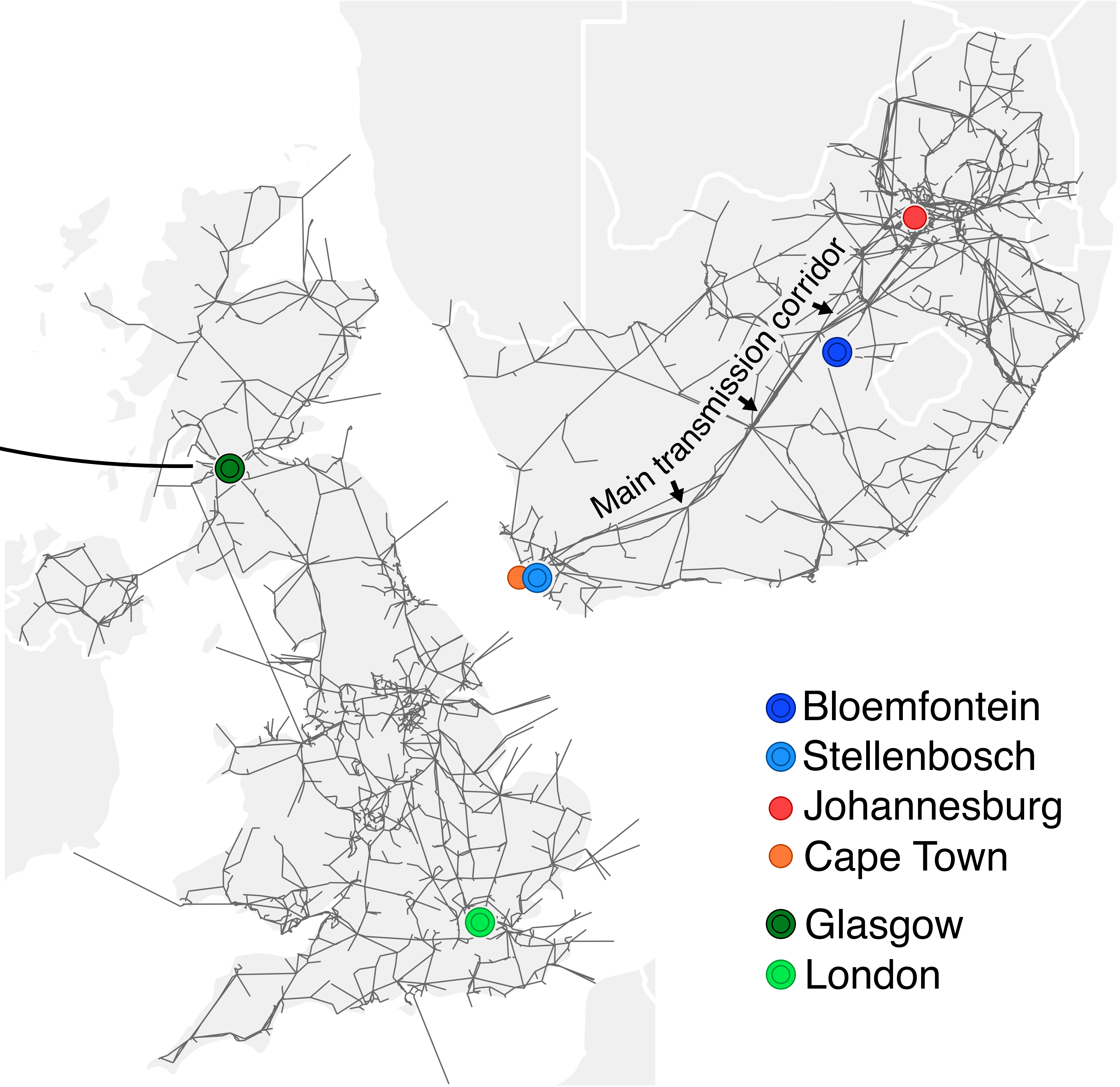
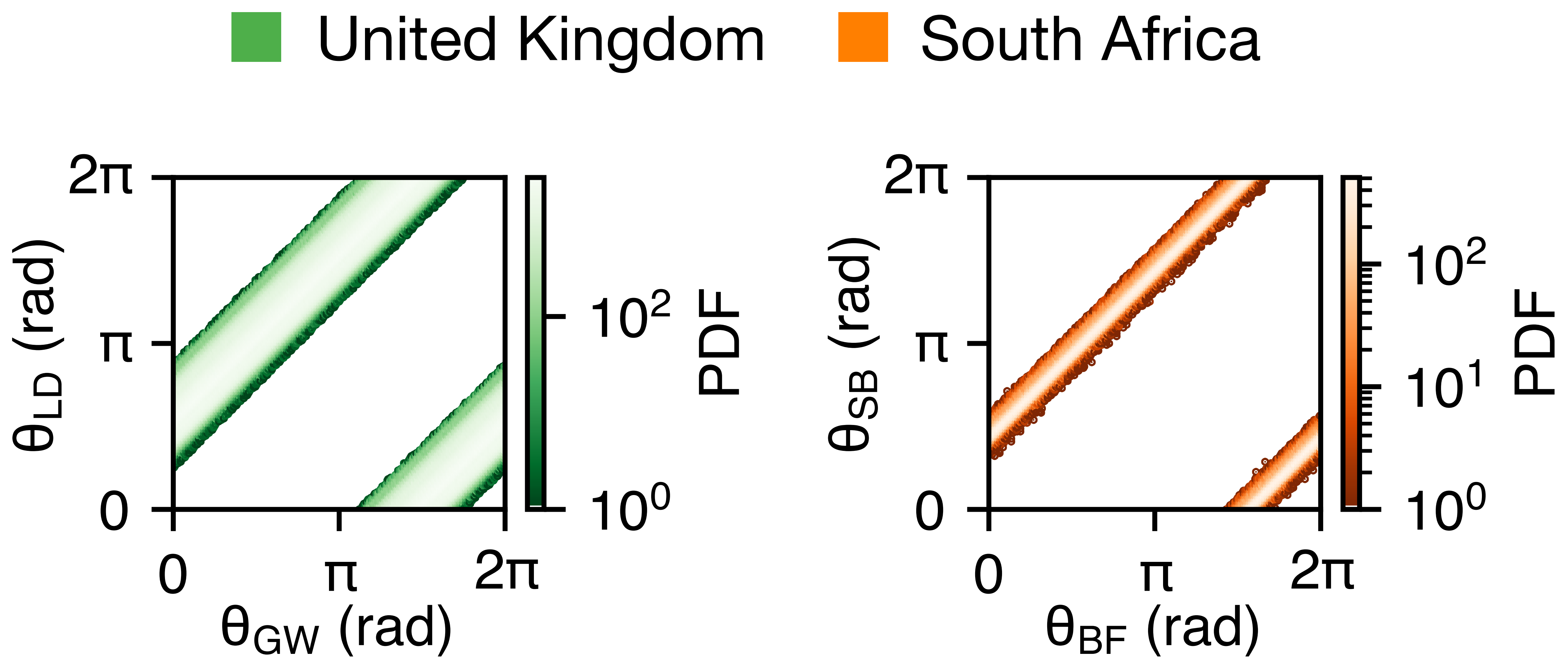
$$\theta_i(t)$$



Phase-locked states



A reduced grid model for phase angles

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \gamma\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\mathrm{d}V}{\mathrm{d}x} = \epsilon\xi$$

$$V(x) = -\delta x - 2\kappa\cos x$$

$$N = 2!$$

$$x(t) = \theta_{\mathrm{LD}}(t) - \theta_{\mathrm{GW}}(t)$$

$$x(t) = \theta_{\mathrm{SB}}(t) - \theta_{\mathrm{BF}}(t)$$

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$$\epsilon > 0, \xi \sim \mathcal{N}(0,1)$$

$$\gamma > 0$$

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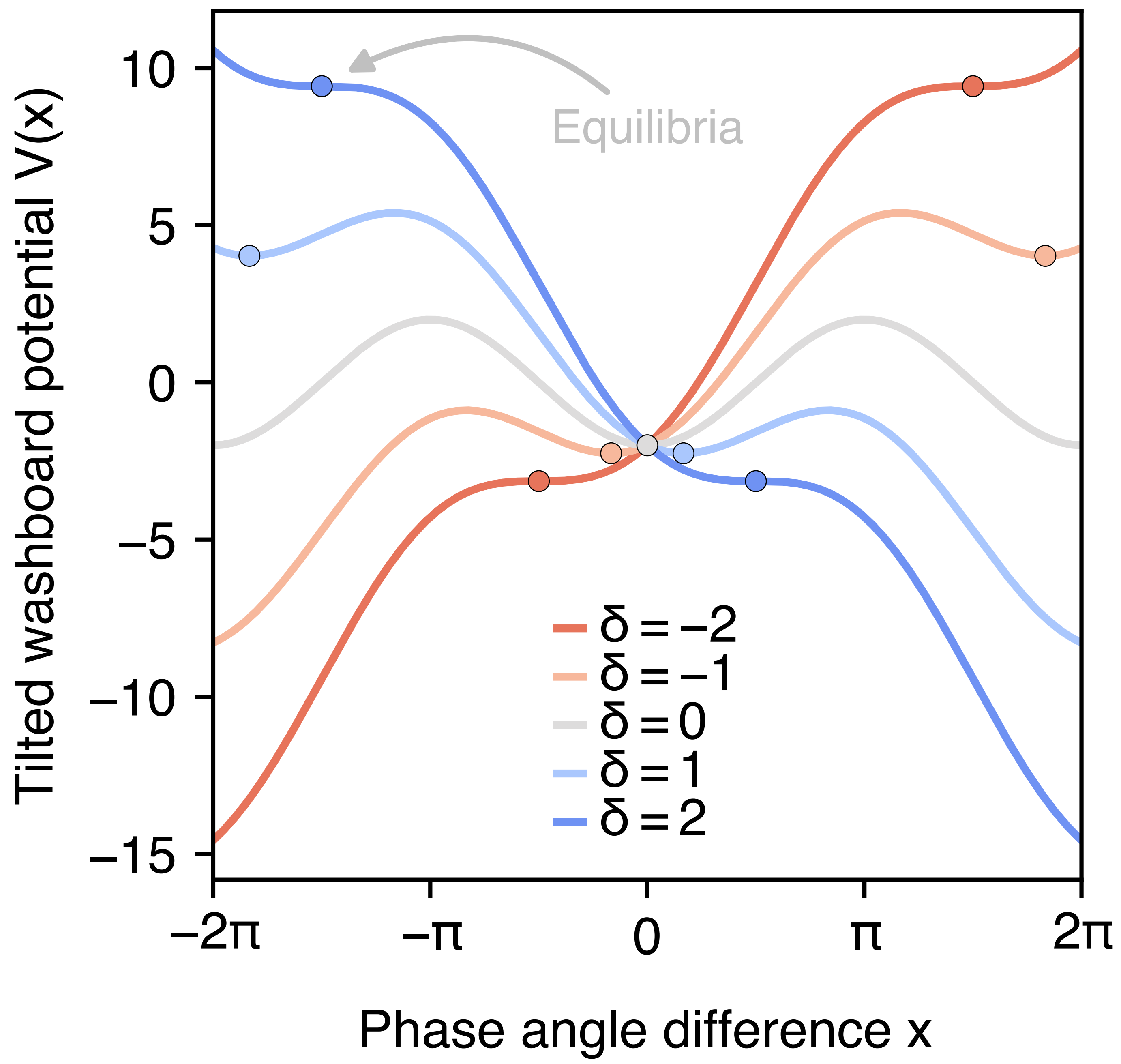
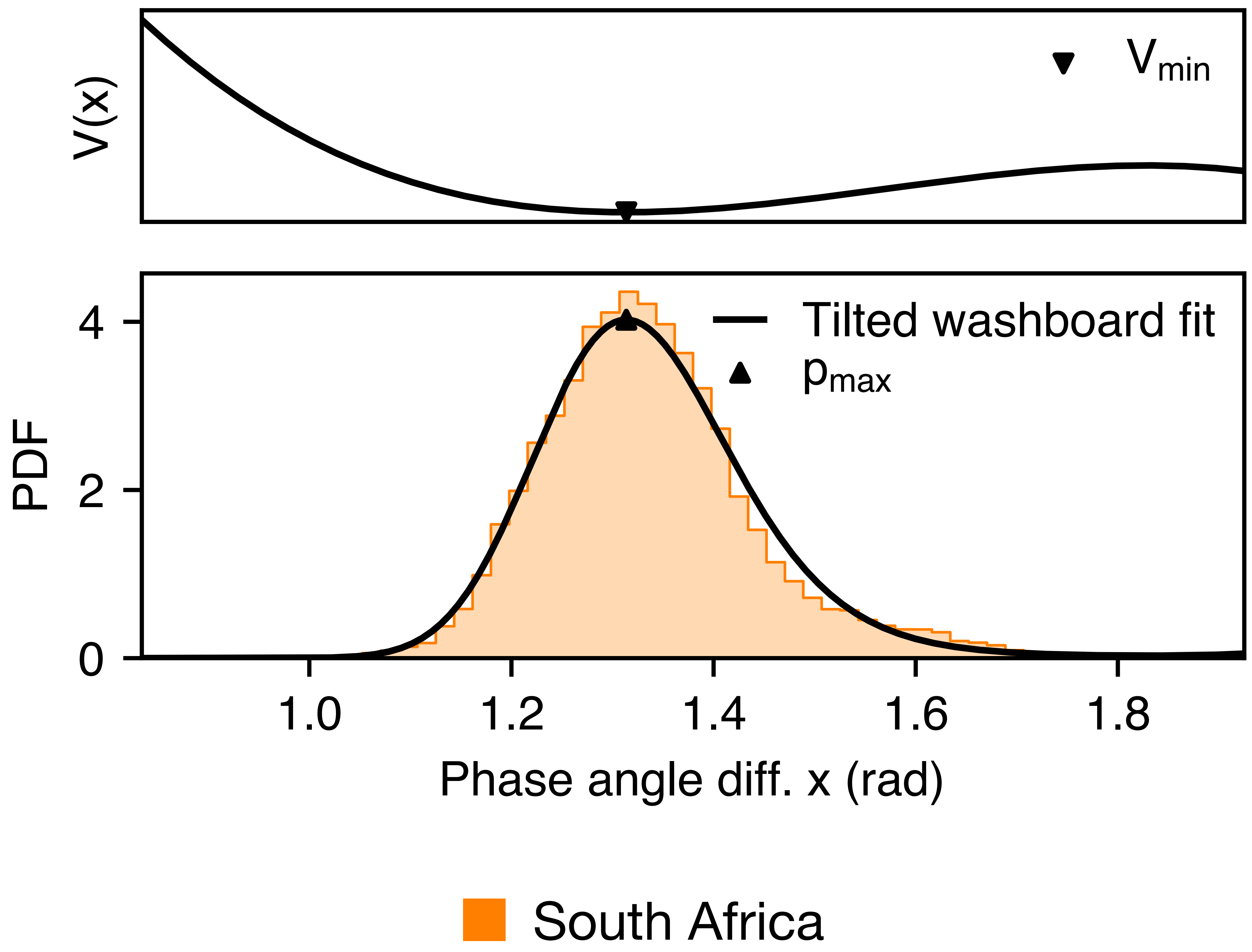


$$p(x) \propto e^{-\gamma V(x)/\epsilon^2} \qquad V(x) = -\delta x - 2\kappa \cos x$$



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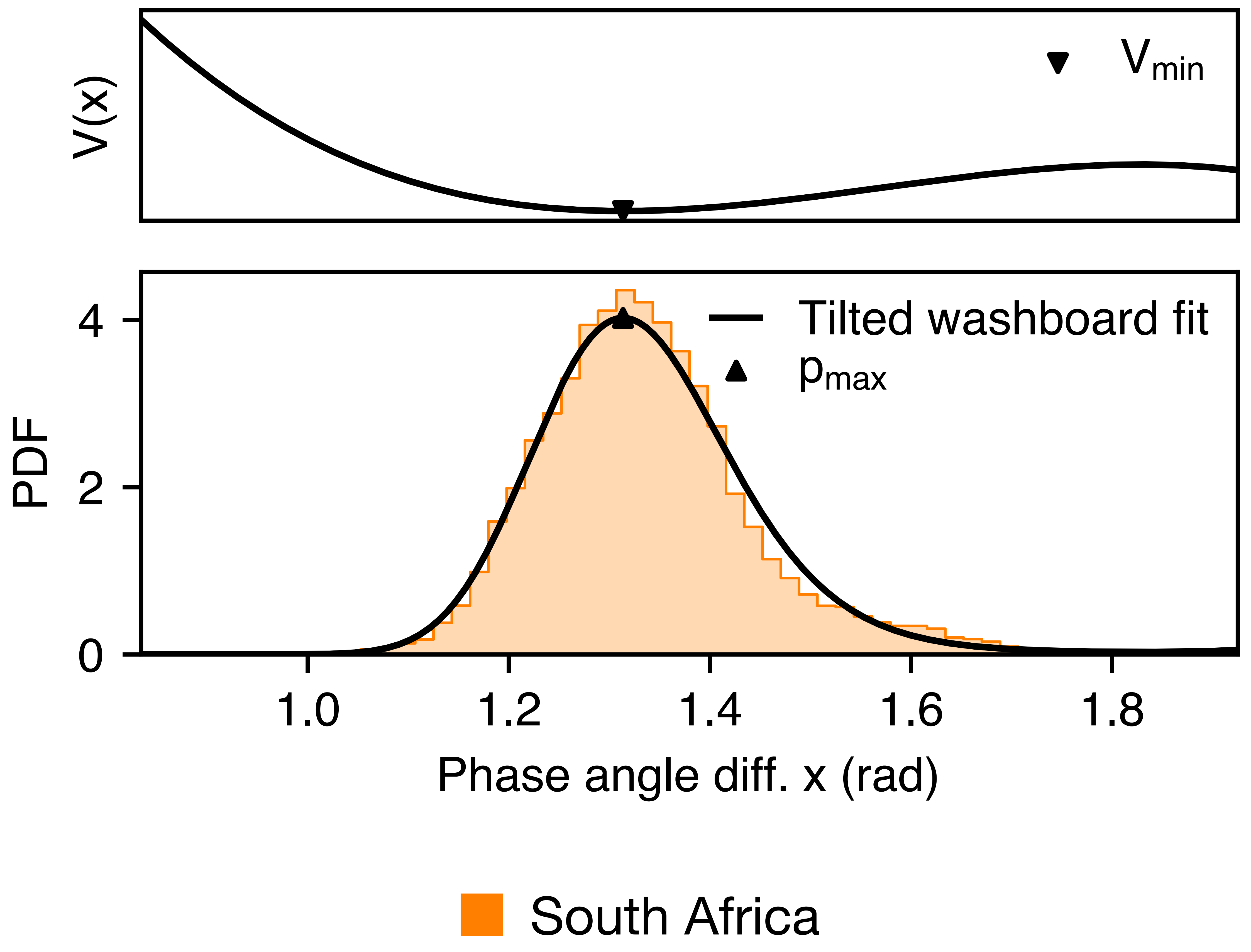

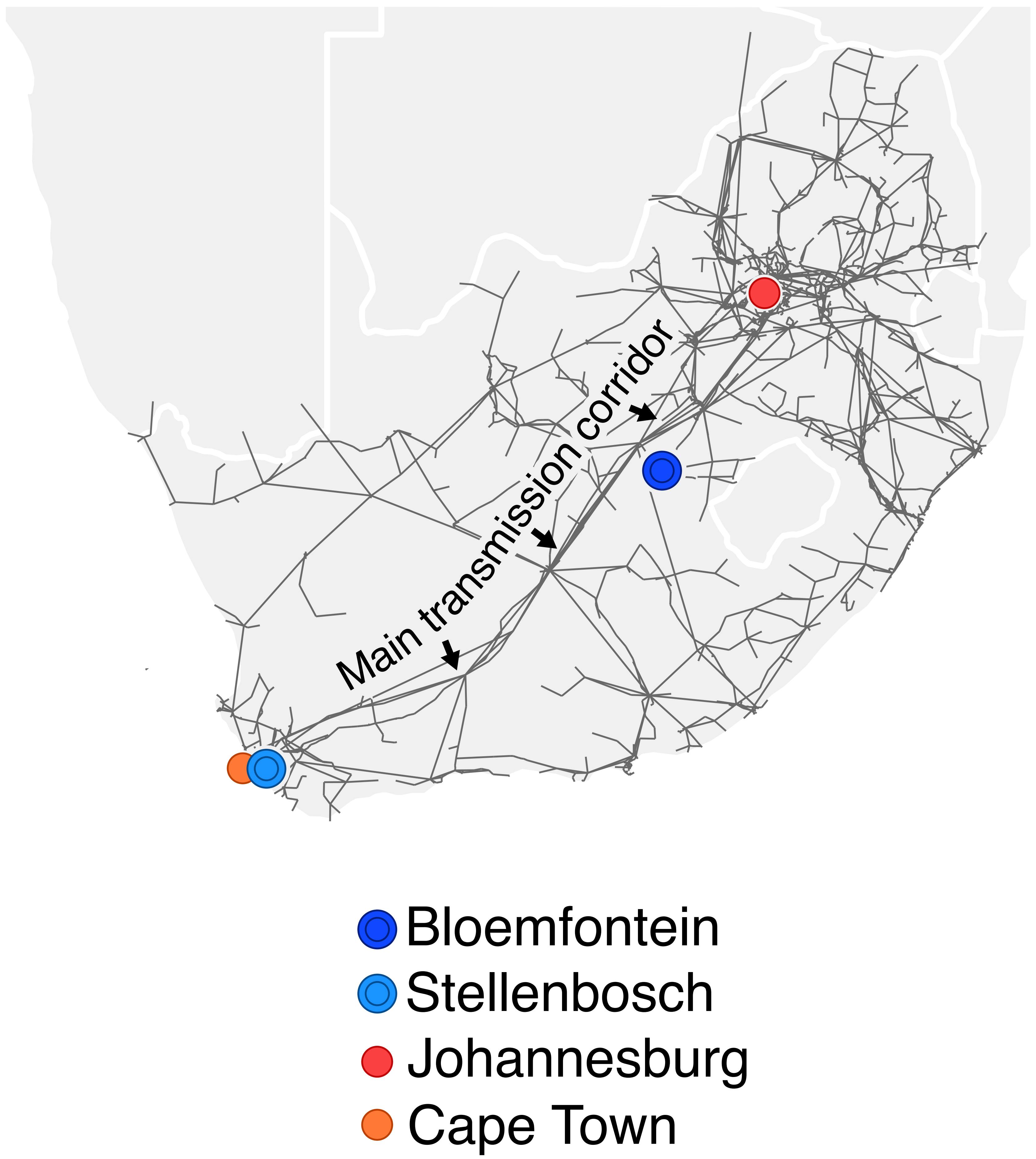

Take home message

Let's look back at the broad scope of the talk

- Show that statistical physics is a helpful tool for describing the emergent properties of power grids

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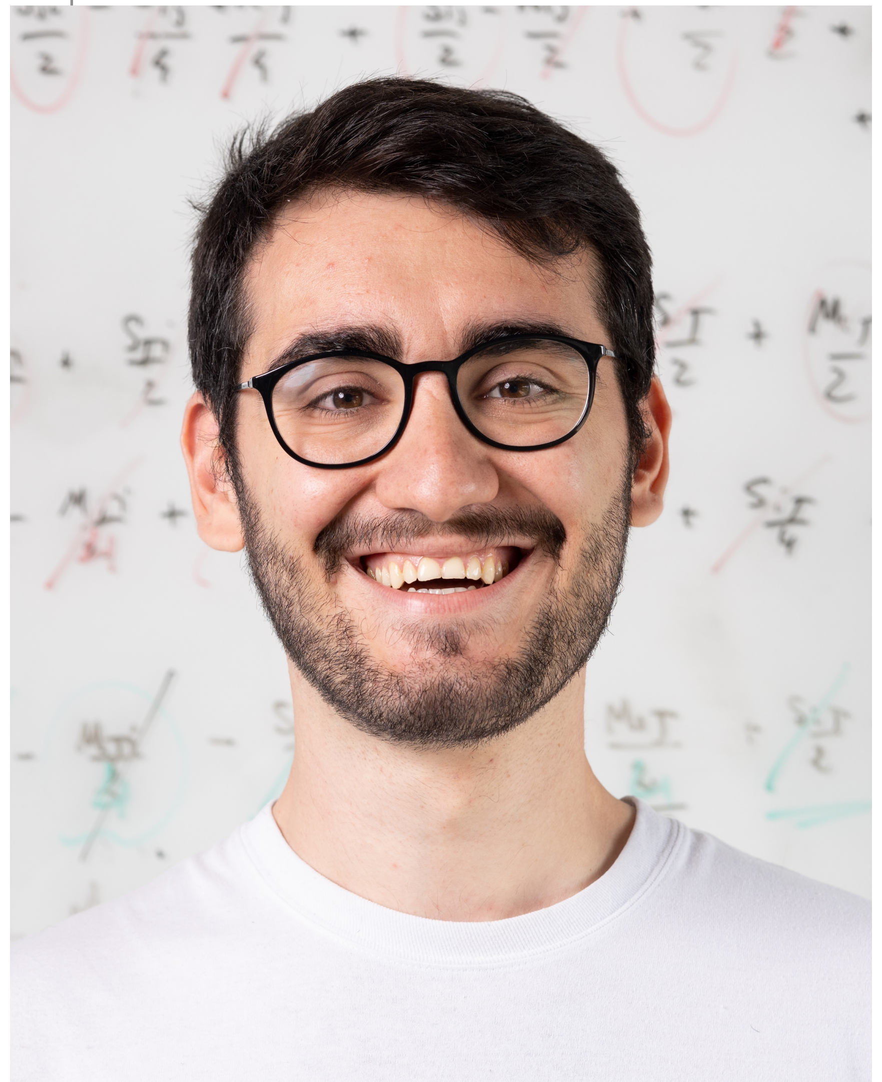

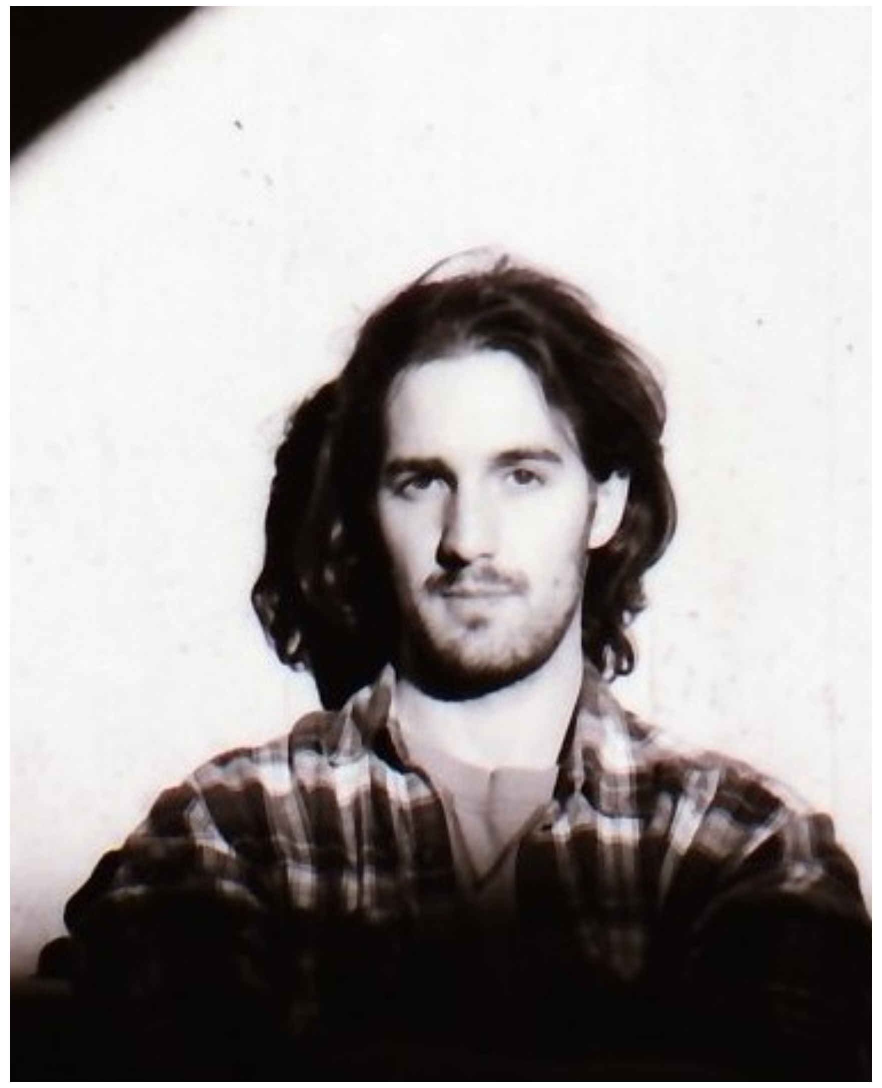

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