Understanding the complexity of frequency and phase angle fluctuations in power grids

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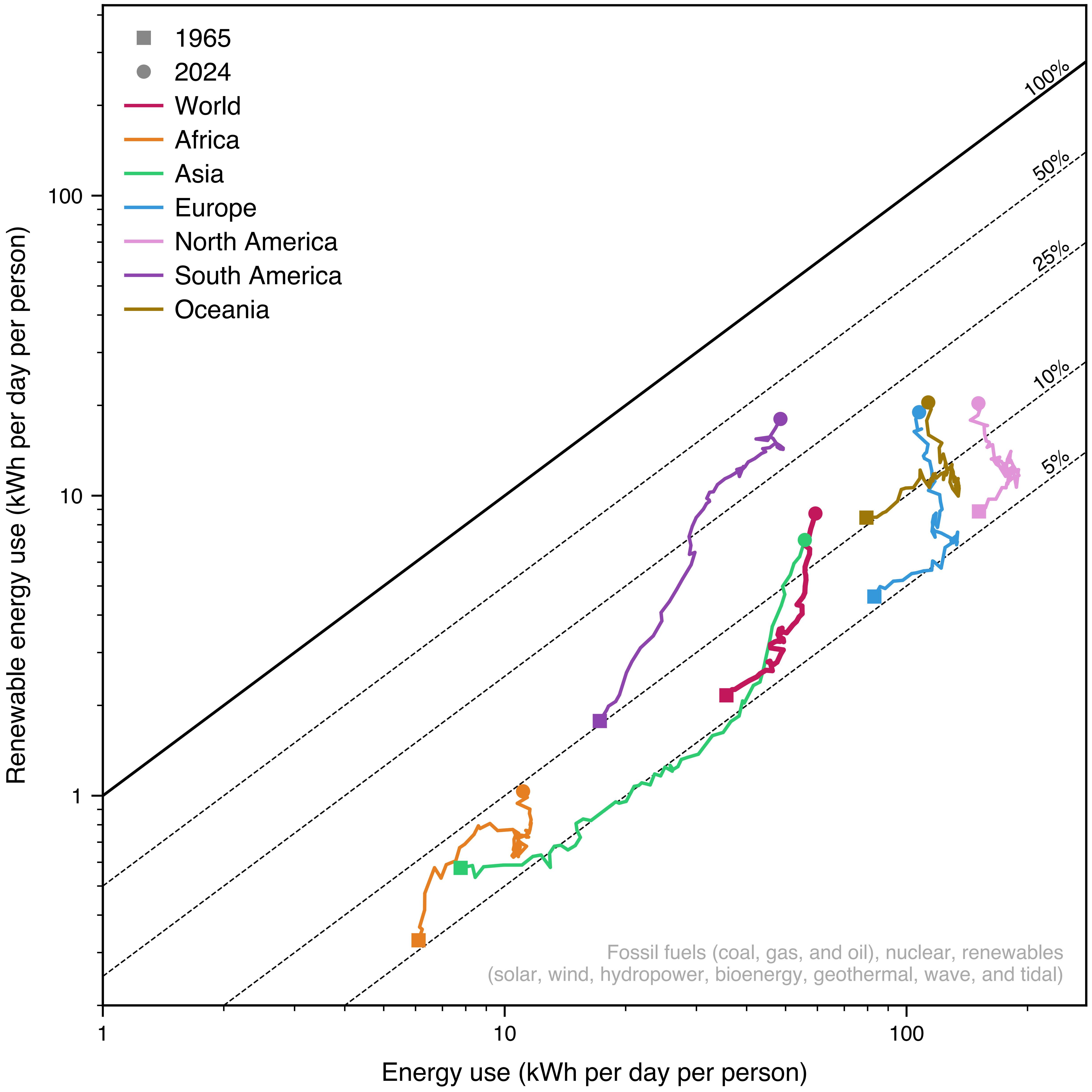
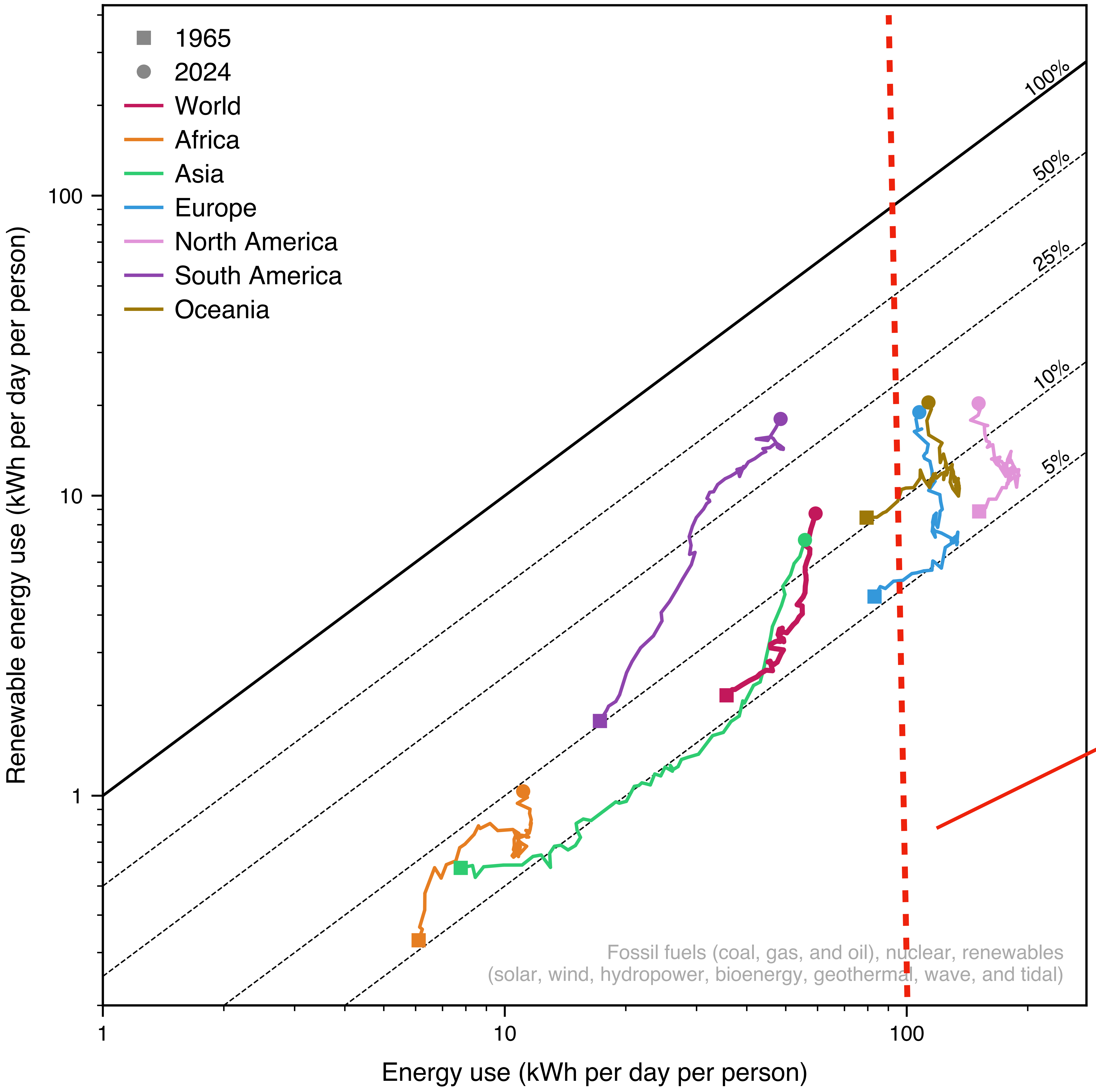
aleable.github.io

Why does it matter?



Data from Global Change Data Lab



Old lightbulb = 100 W power draw

1 LED lightbulb = 10 W power draw

~42 old bulbs continuously on

~420 LED bulbs continuously on

Data from Global Change Data Lab

177 net-zero pathways for the year 2100 project:

- Renewables to be 60% of primary energy supply (today ~14%, matching data shown before)
- Electricity consumption increasing from about 20% to 50%

DeAngelo et al. Nature Communications 2021

My grandparents' stove (< 1940)

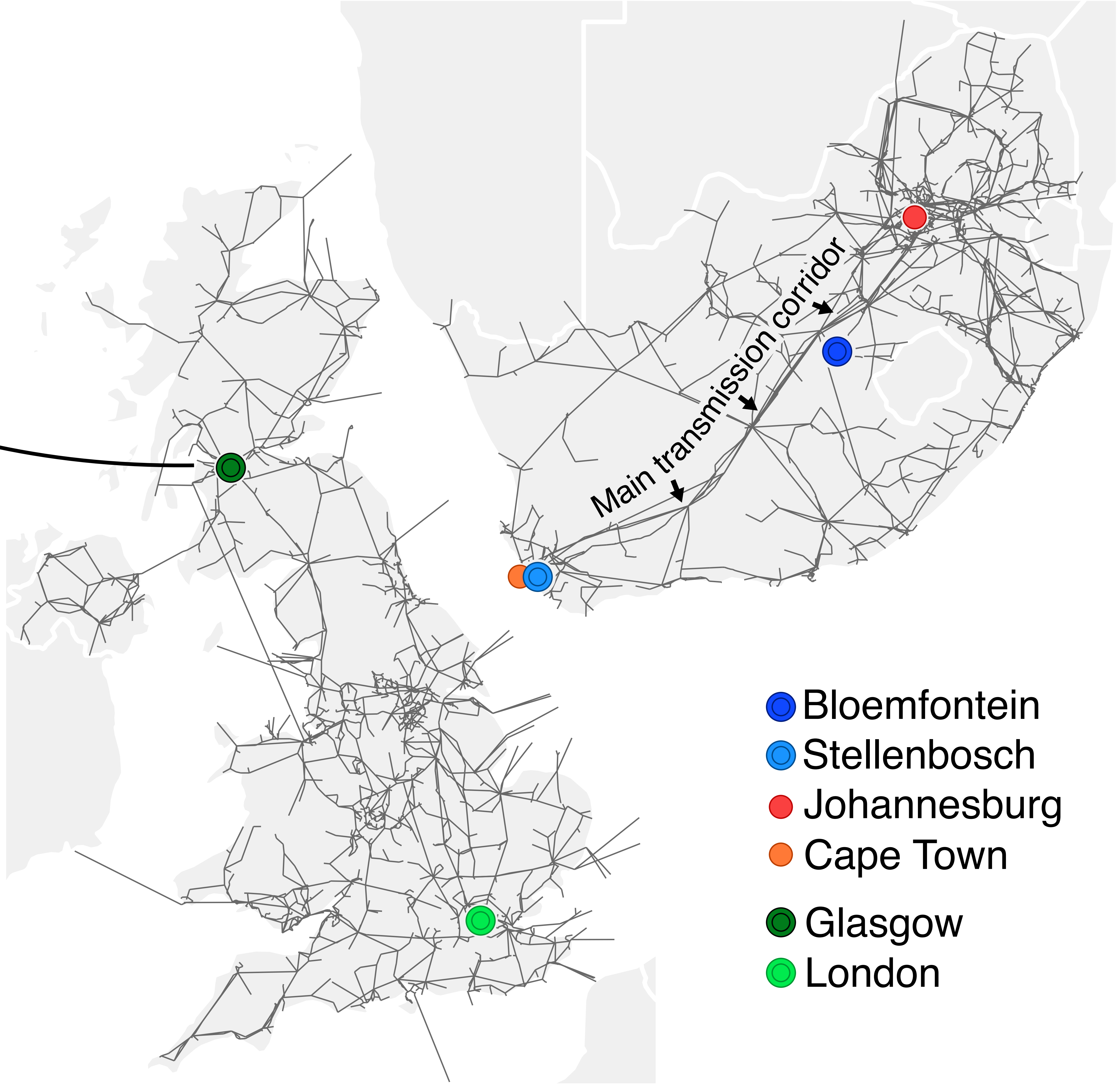
My parents' house in Italy (fireplace built in 1996)

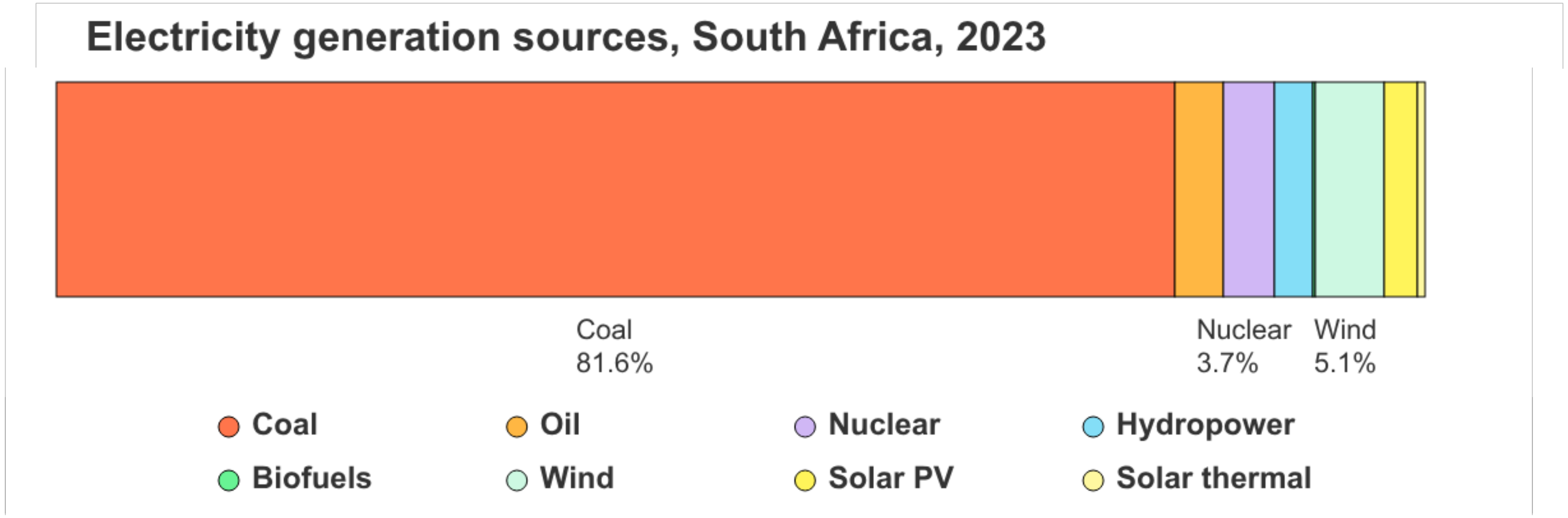
My flat in London in 2026

Goal of today

Show that statistical physics describes power grid measurements

$$\omega(t) = 2\pi\,(f(t) - f_{\mathrm{R}})$$
$$\theta_i(t) \in [0, 2\pi)$$





Data from International Energy Agency

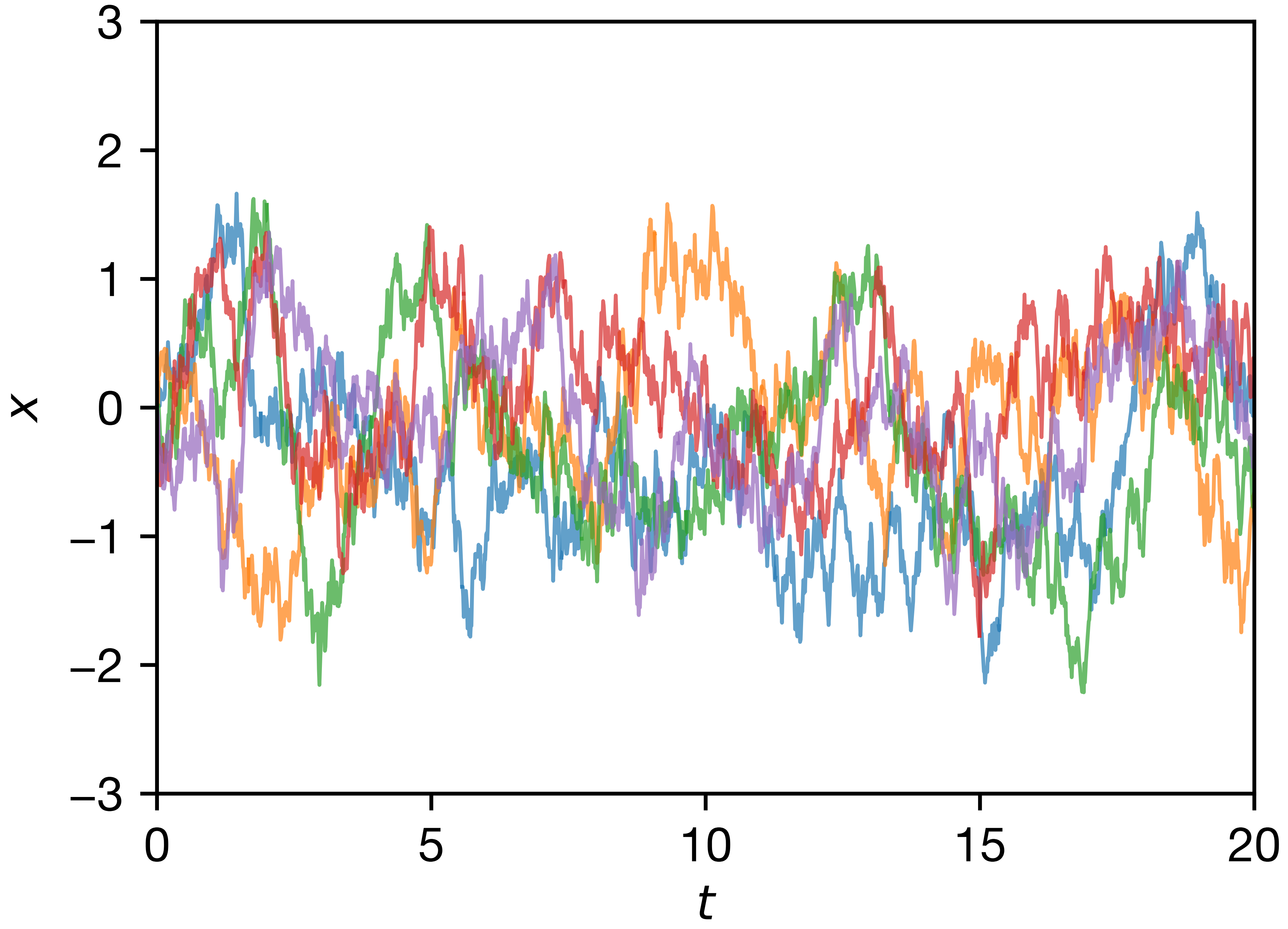
Frequency

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\gamma x + \epsilon\xi$$

$\gamma =$ Damping

$\epsilon\xi =$ Gaussian noise

$\beta := \gamma/\epsilon^2$



$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\gamma x + \epsilon\xi$$

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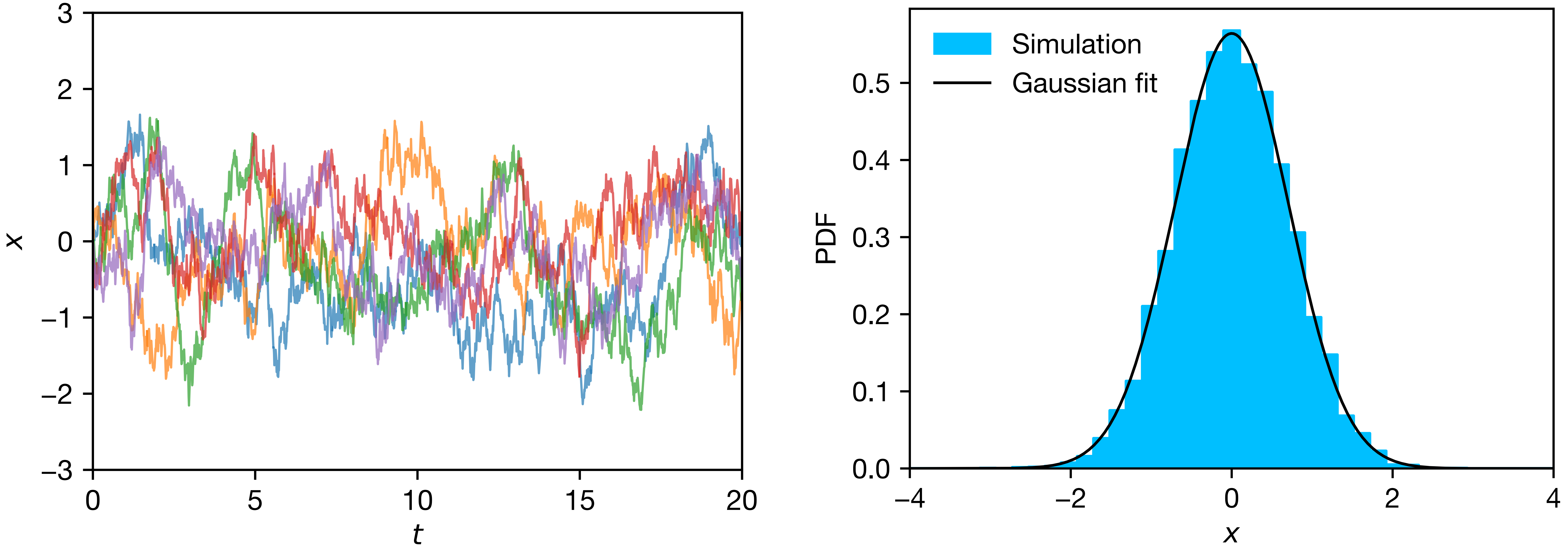
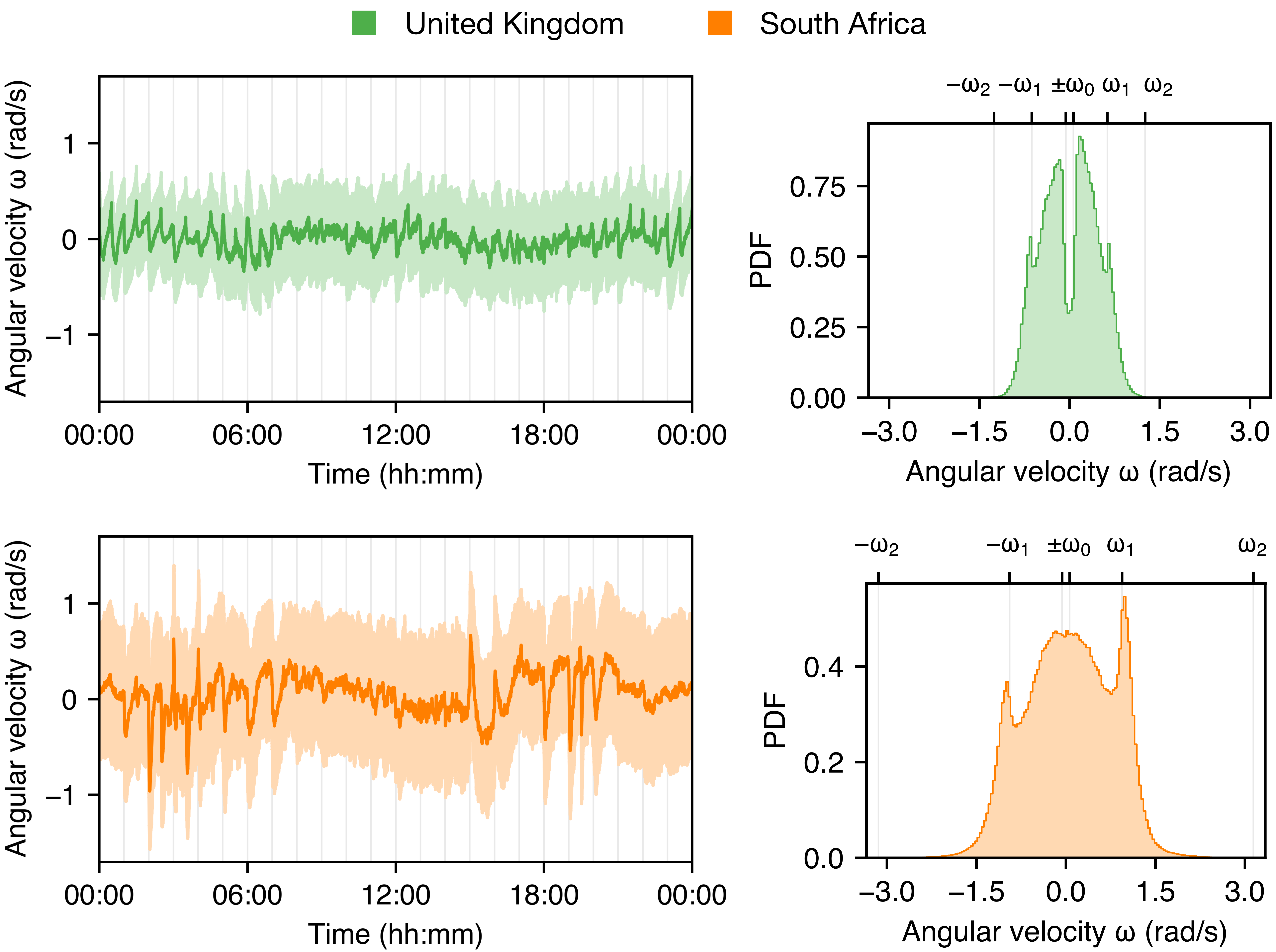
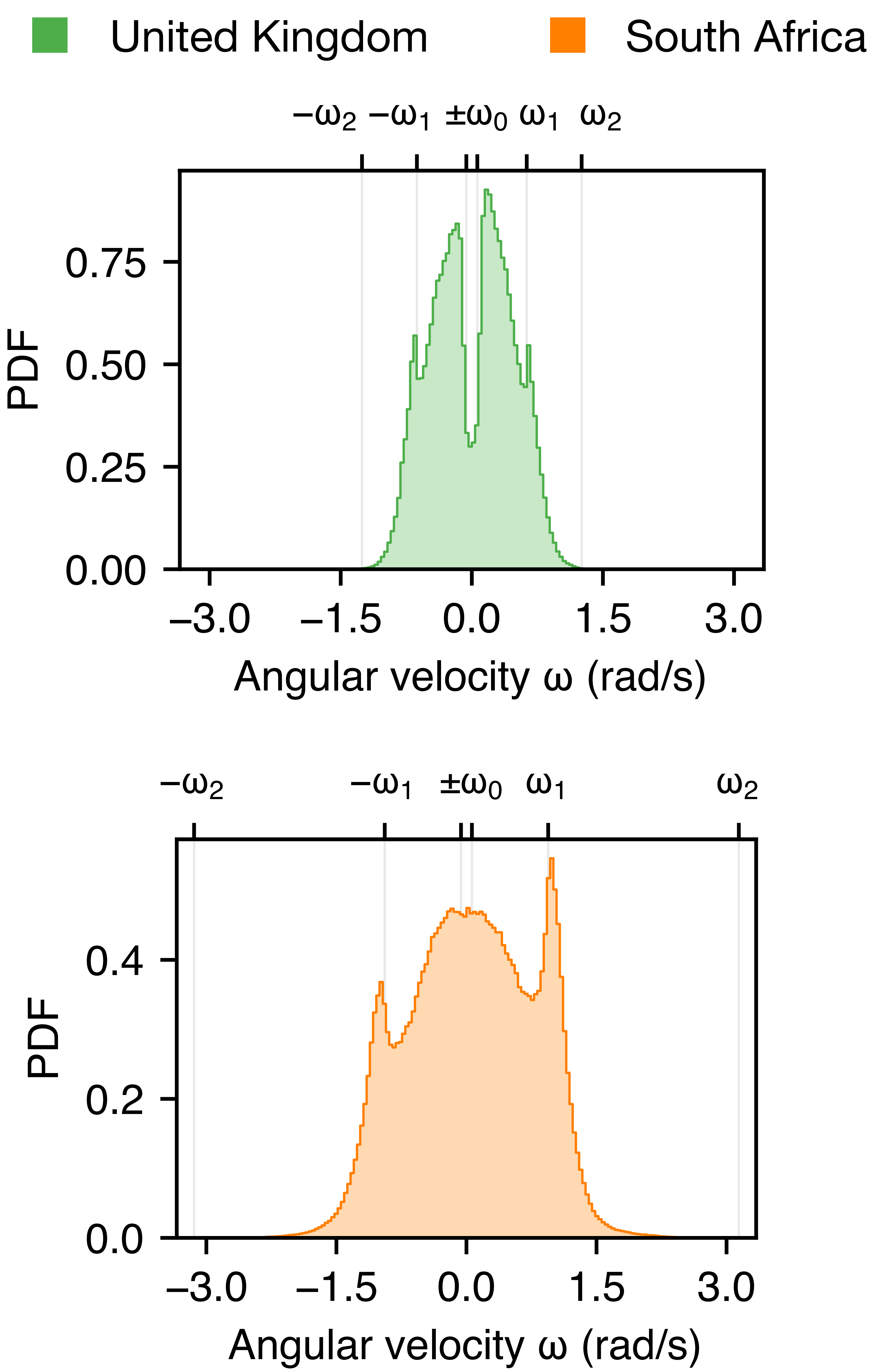
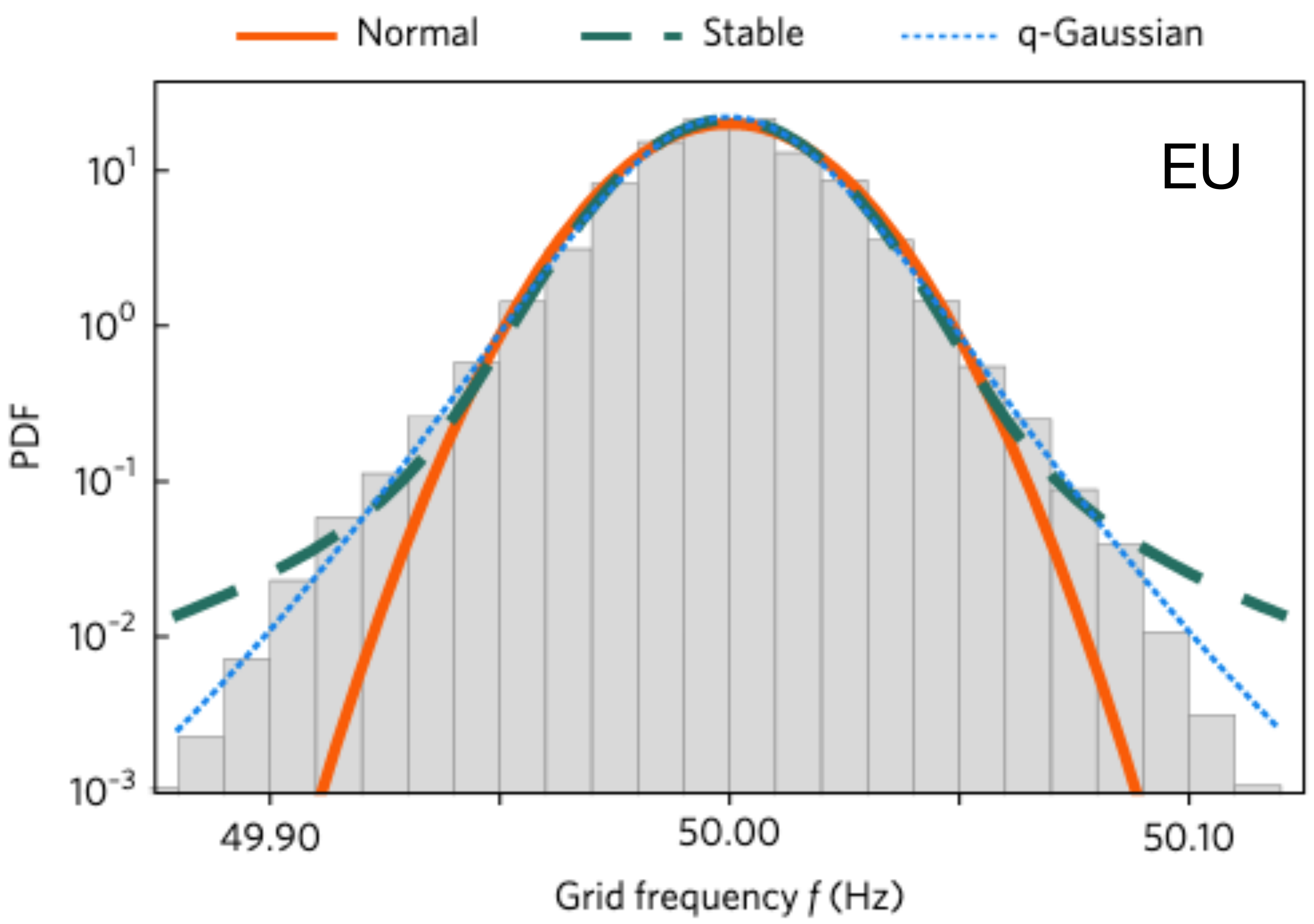
$$p(x) \propto e^{-\beta x^2}$$



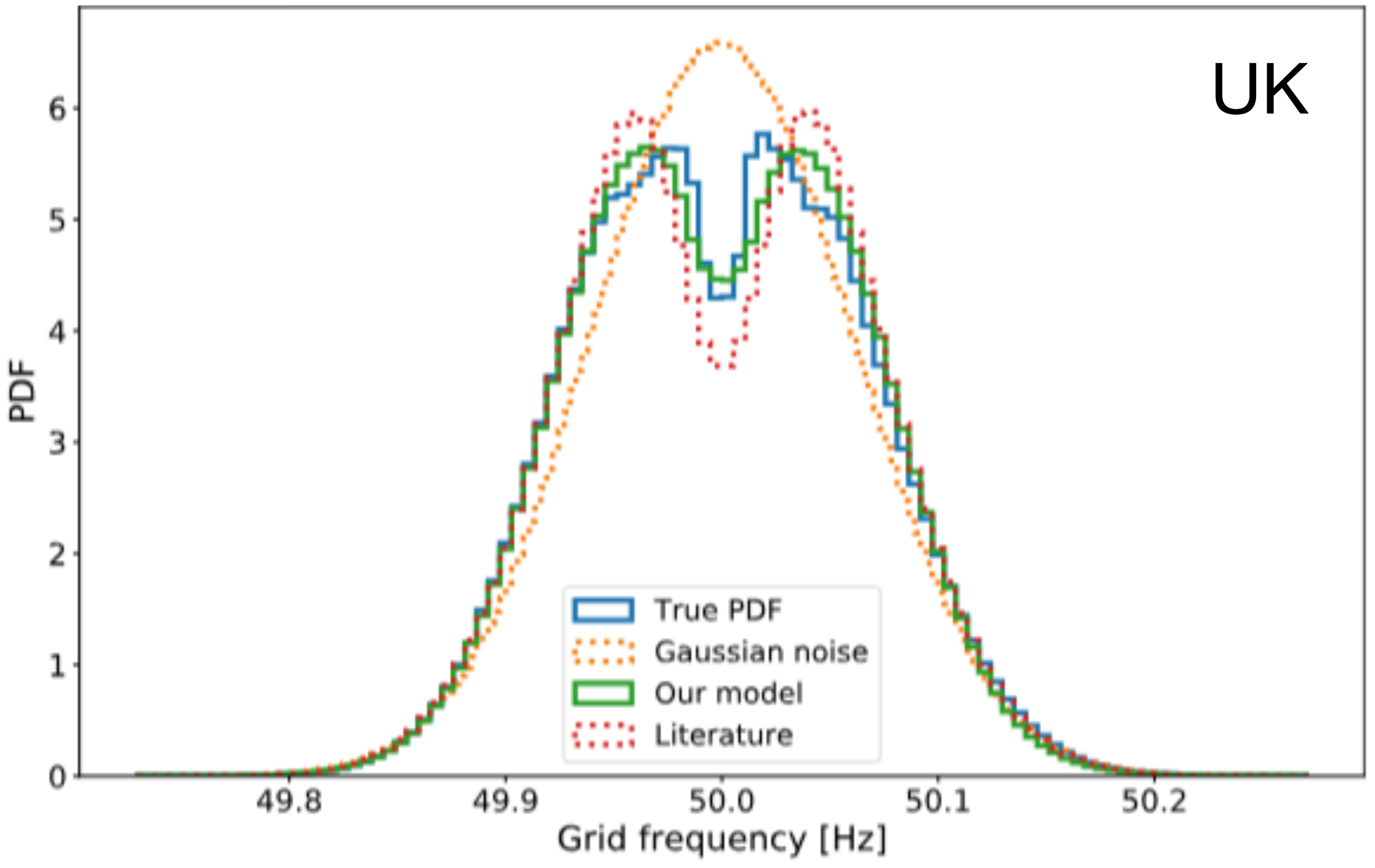


United Kingdom
South Africa
Angular velocity ω (rad/s)
Time (hh:mm)
00:00
06:00
12:00
18:00
00:00
1
0
−1
PDF
0.75
0.50
0.25
0.00
−3.0
−1.5
0.0
1.5
3.0
−ω2 −ω1 ±ω0 ω1 ω2
0.4
0.2
0.0





Schäfer et al. Nature Energy 2018



Kraljic IEEE Transactions on Power Systems 2023

Superstatistics

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = H(\omega) + P(t) + \epsilon\xi$$

$$\frac{dx}{dt} = -\gamma x + \epsilon\xi$$

$$\beta := \gamma/\epsilon^2$$

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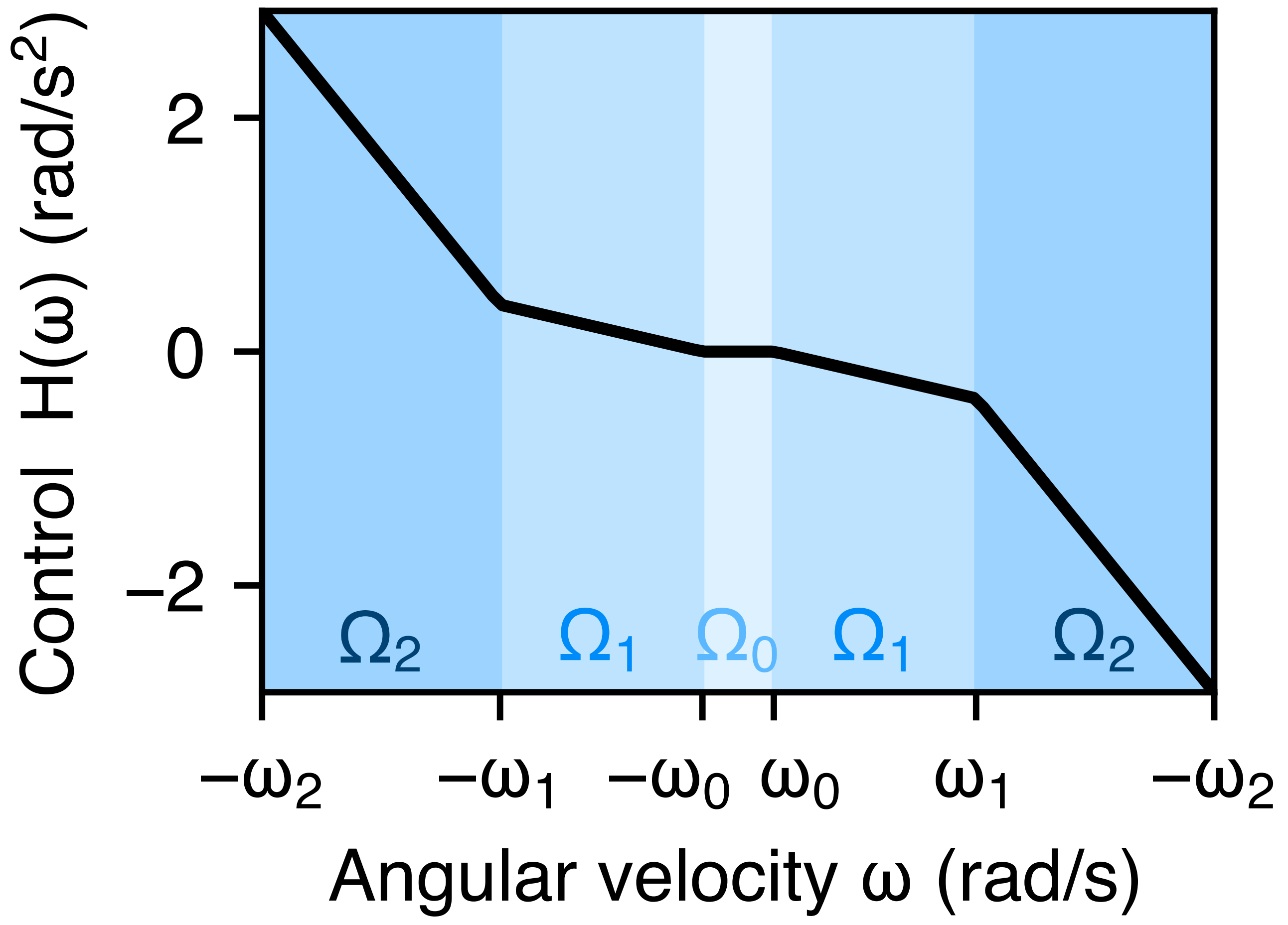
- Nonlinear control



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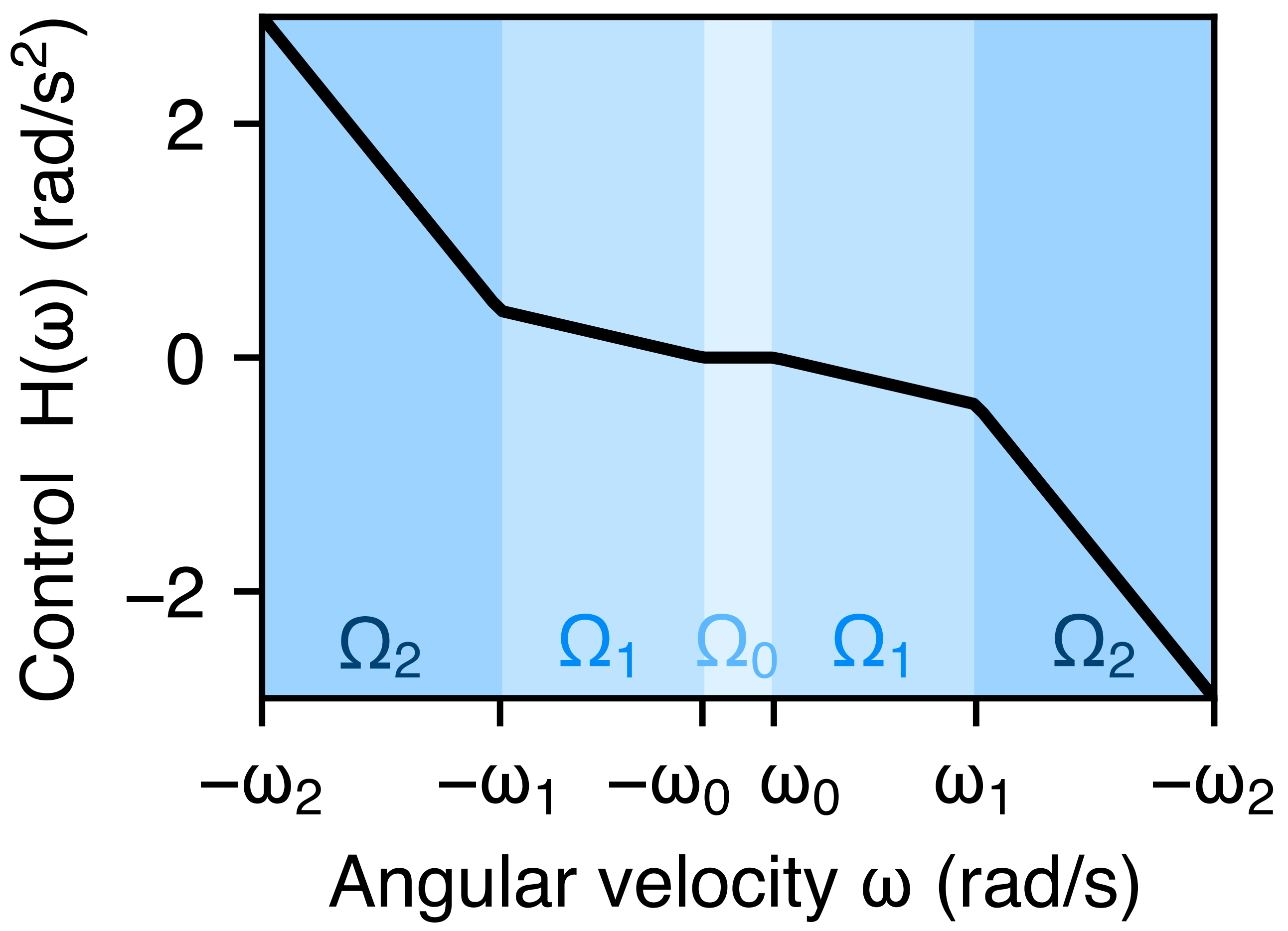

- $P(t)$ is the **slow** superstatistical variable

Superstatistics

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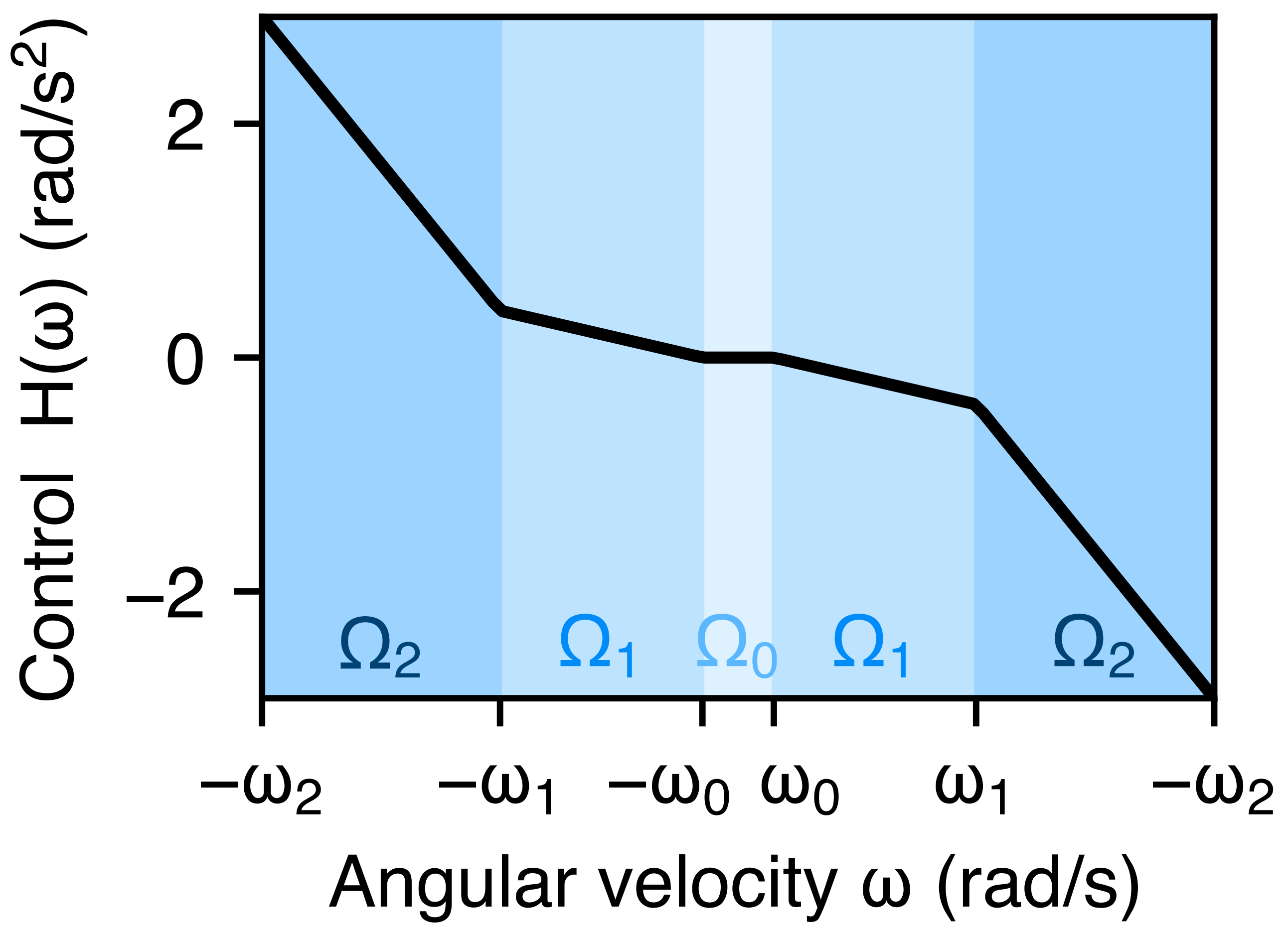
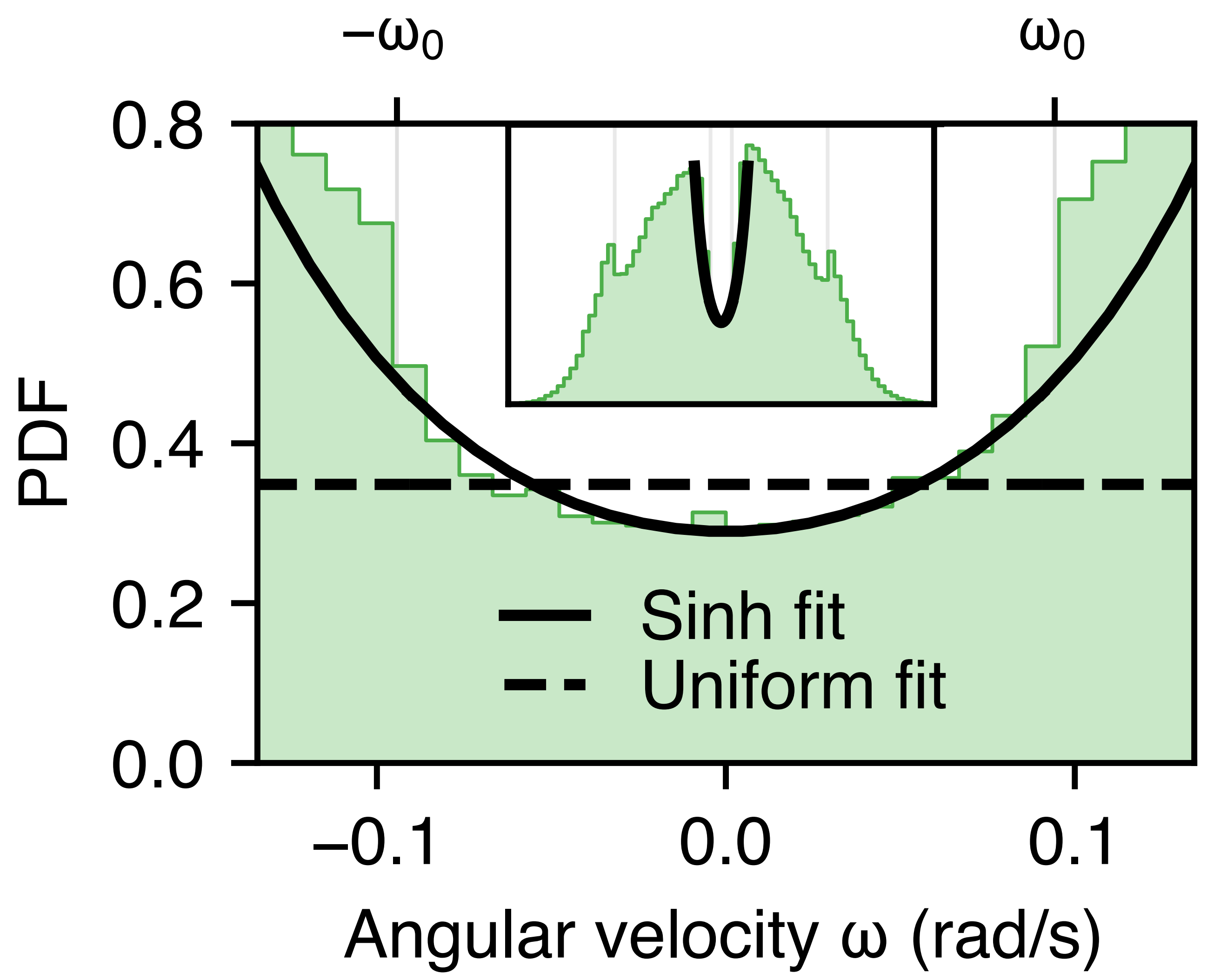


$$\longrightarrow \quad p(\omega) \propto \int f(\omega \mid P)\, \varphi(P)\, \mathrm{d}P$$

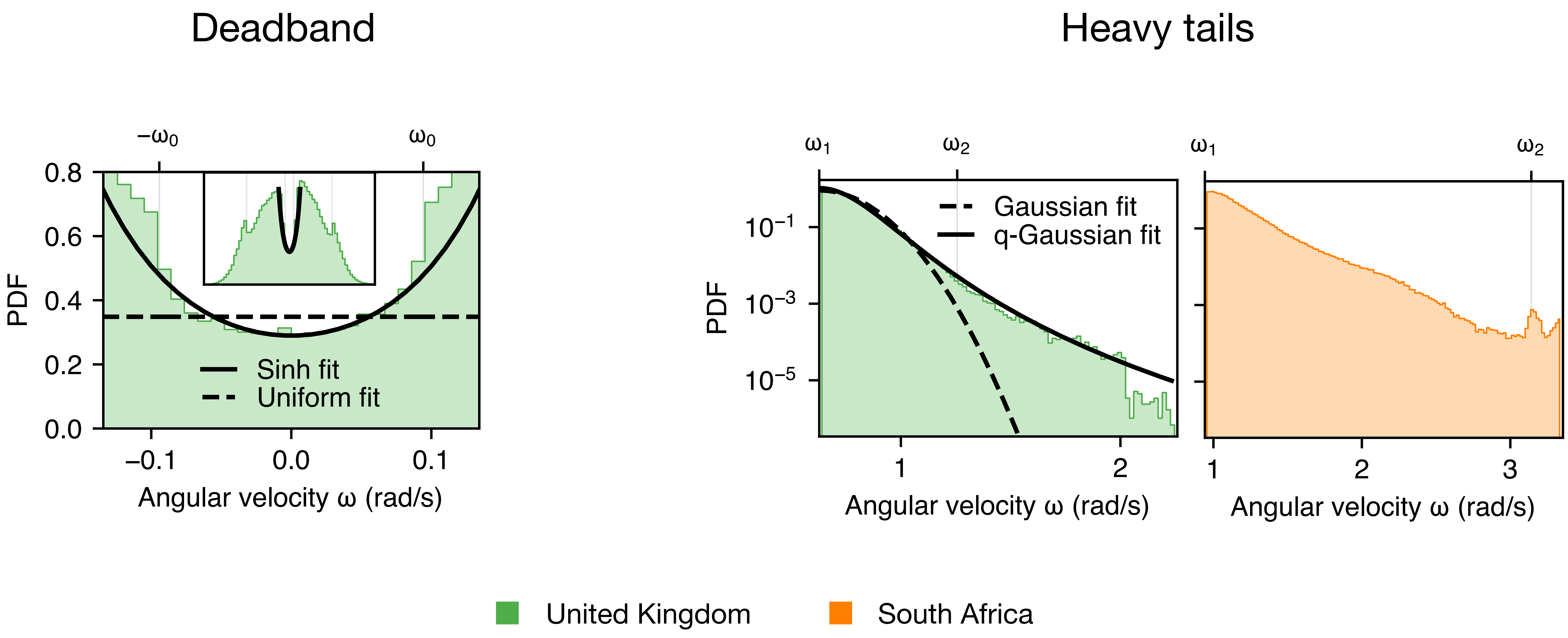
C. Beck Physical Review Letters 2001

- $P(t)$ is the **slow** superstatistical variable

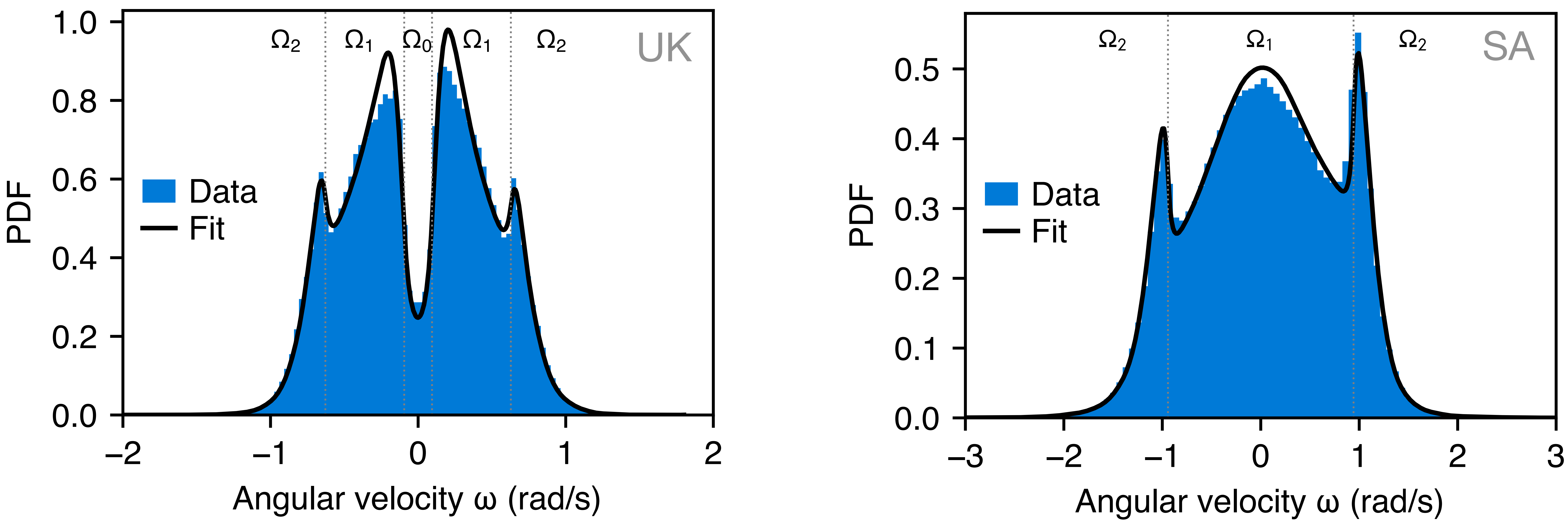
Deadband



United Kingdom

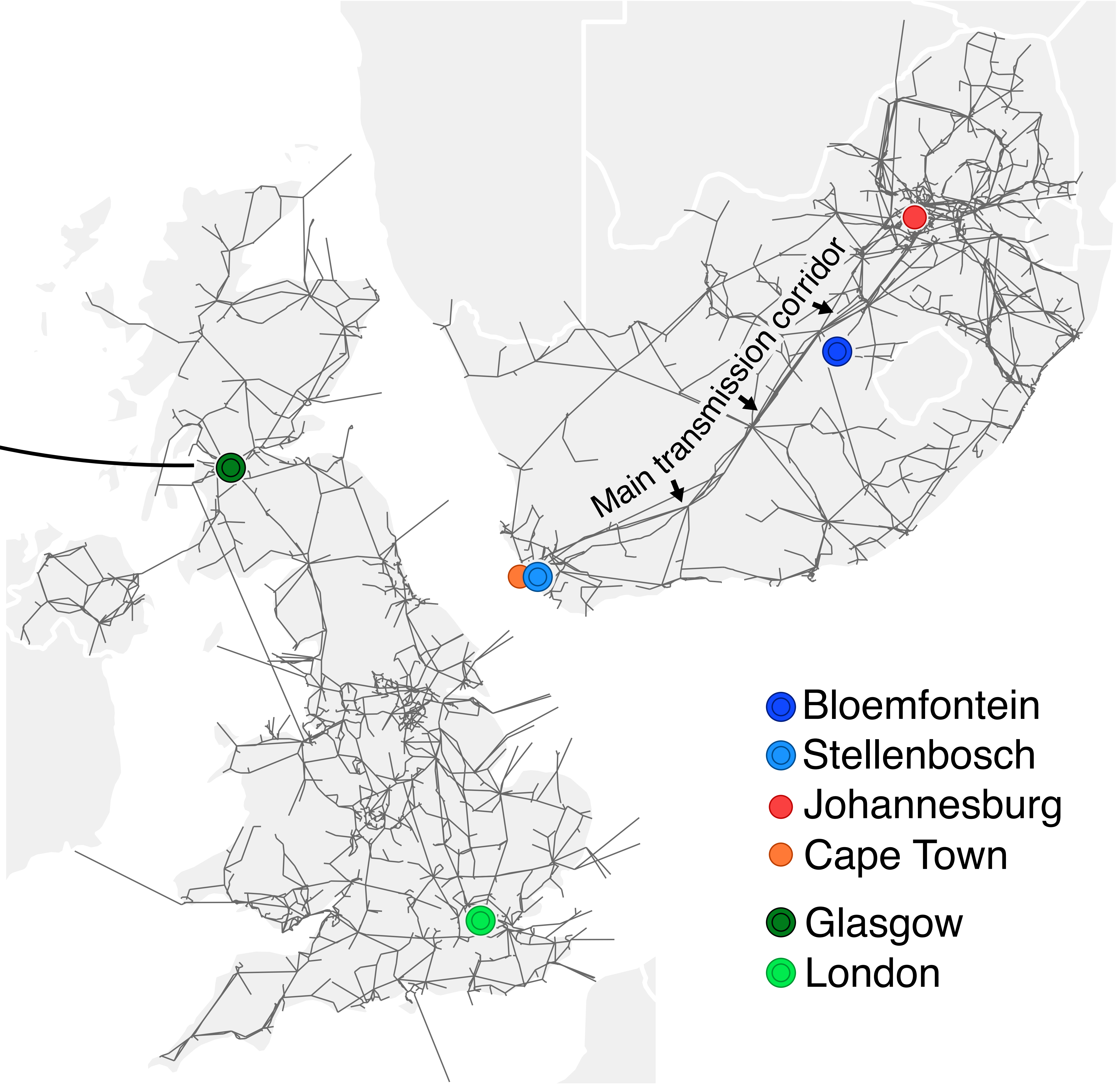

Deadband
Heavy tails
$-\omega_0$
ω_0
0.8
0.6
0.4
0.2
0.0
PDF
Sinh fit
Uniform fit
−0.1
0.0
0.1
Angular velocity ω (rad/s)
ω_1
ω_2
10^{-1}
10^{-3}
10^{-5}
Gaussian fit
q-Gaussian fit
1
2
3
United Kingdom
South Africa

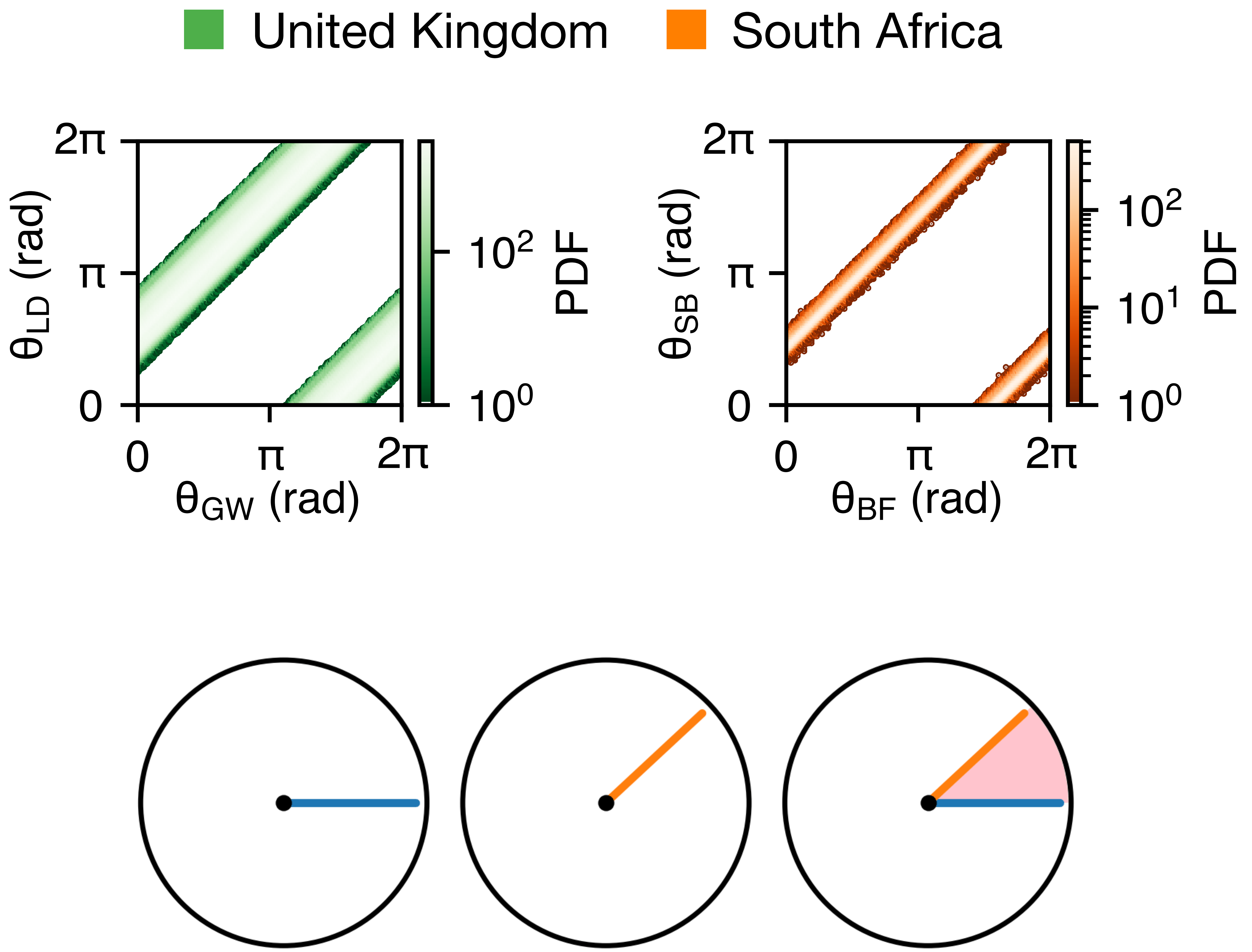
From the same idea: an algorithm for statistical inference



Phase angles

$$\omega(t) = 2\pi\,(f(t) - f_{\mathrm{R}})$$
$$\theta_i(t) \in [0,2\pi)$$





United Kingdom
South Africa
2π
π
0
θLD (rad)
θGW (rad)
10^0
10^2
PDF
θSB (rad)
θBF (rad)
10^1

Mathematical details

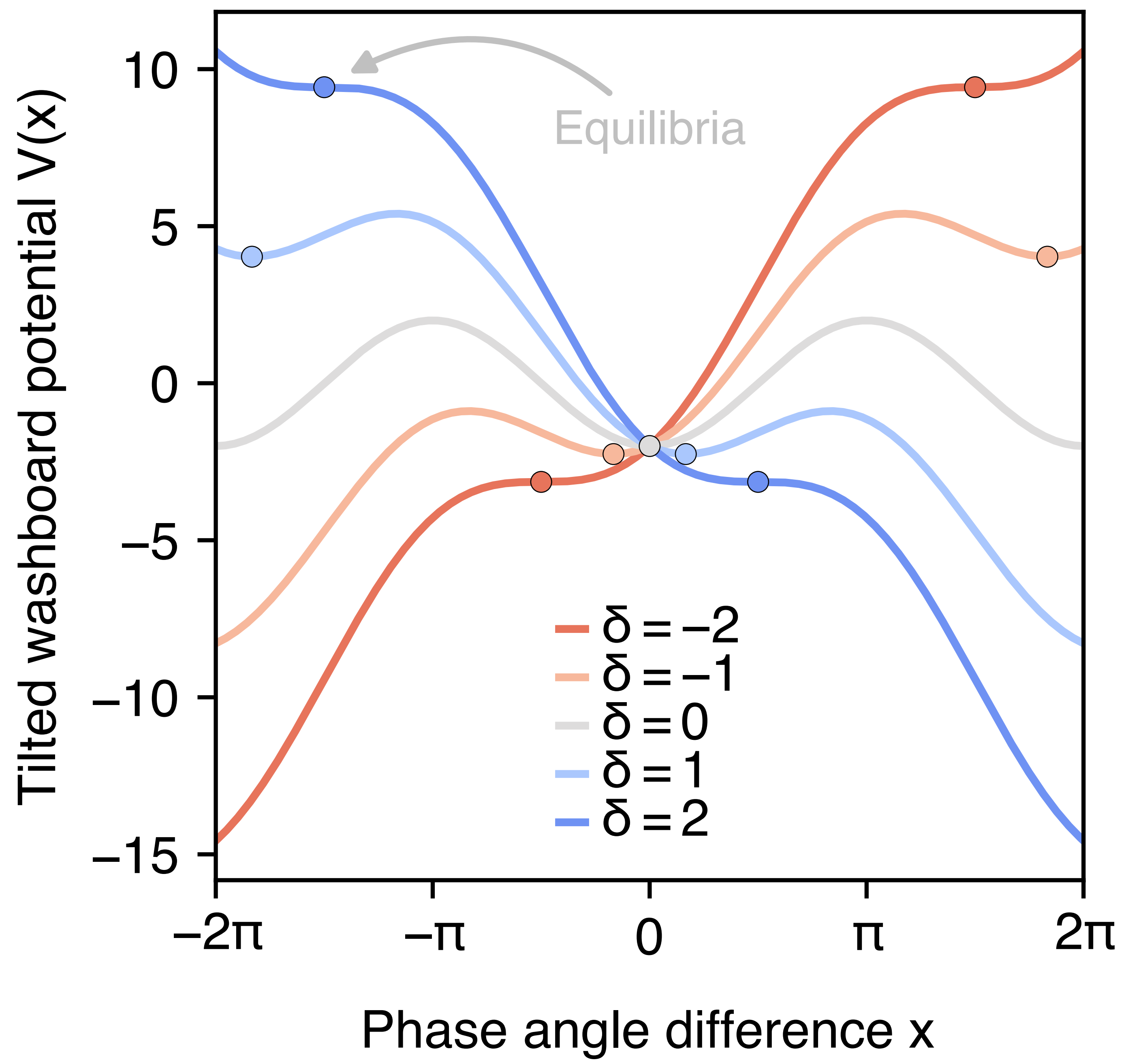
$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \frac{dV}{dx} = \epsilon\xi$$

$N = 2!$

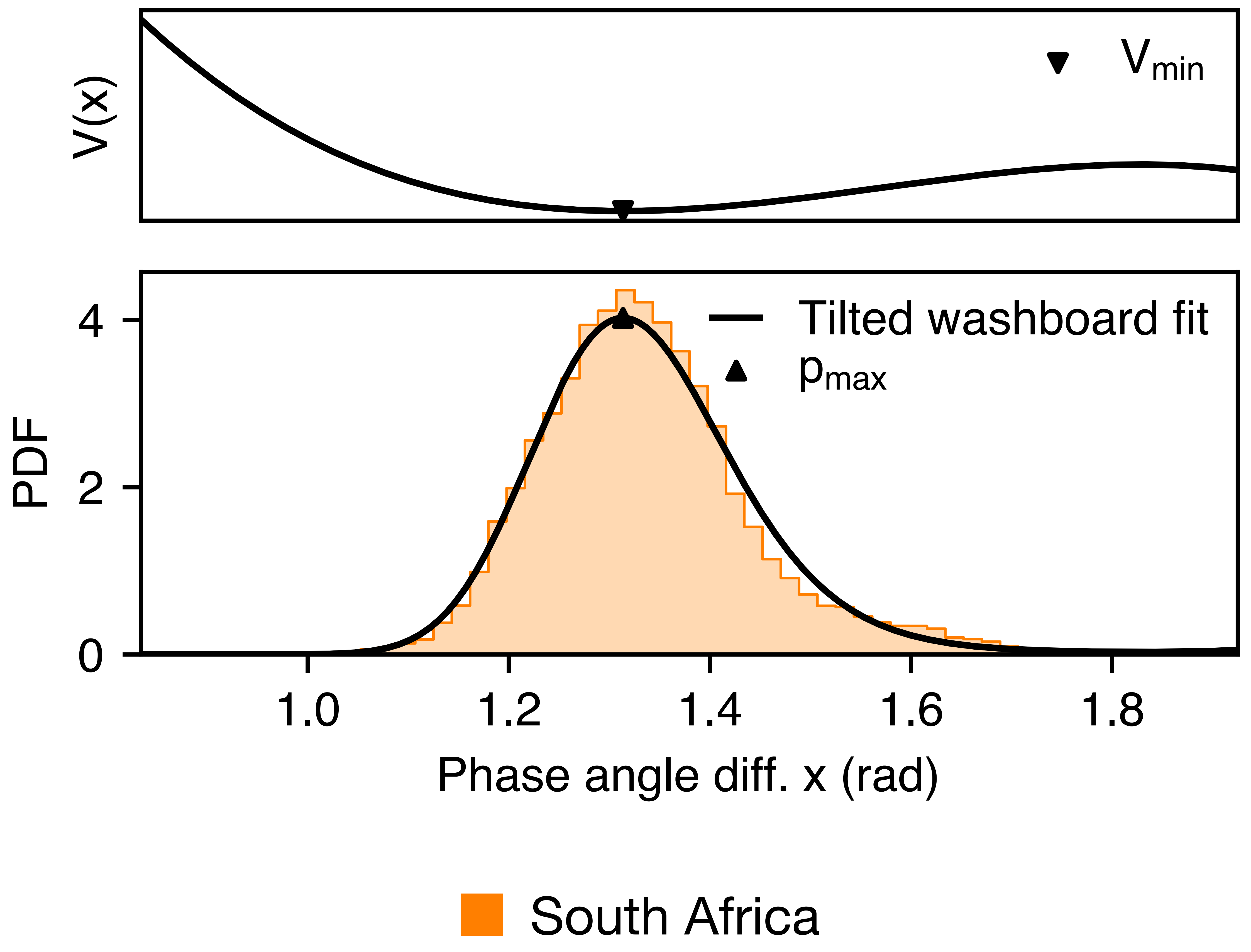
$$x(t) = \theta_{LD}(t) - \theta_{GW}(t)$$

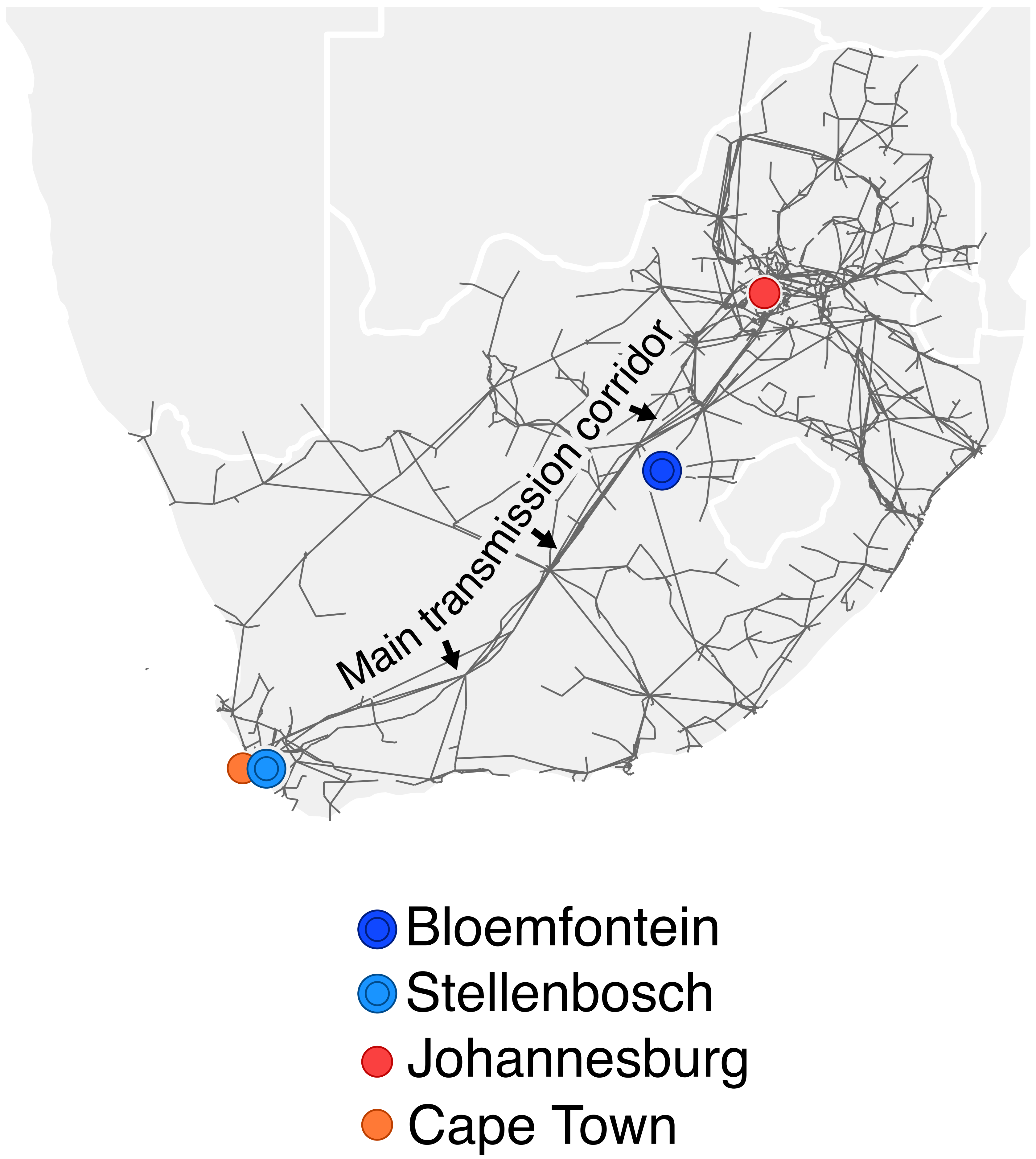
$$x(t) = \theta_{SB}(t) - \theta_{BF}(t)$$

- $V(x) = -\delta x - 2\kappa\cos x$
- $\epsilon\xi =$ Gaussian noise
- $\gamma =$ Damping



$$p(x) \propto e^{-\gamma V(x)/\epsilon^2} \qquad V(x) = -\,\delta x - 2\kappa \cos x$$





- Understanding the complexity of frequency and phase angle fluctuations in power grids

Preprint	arXiv:2604.03133
Code (GitHub)	aleable/power-grid-complexity
Data (Zenodo)	10.5281/zenodo.19397526
Slides	aleable.github.io/talks.html

- Stochastic Modeling of Power-Grid Frequency Fluctuations in Low-Inertia Systems via a Gaussian-Core Potential and Superstatistics (ICPET 2026 Chengdu, China)

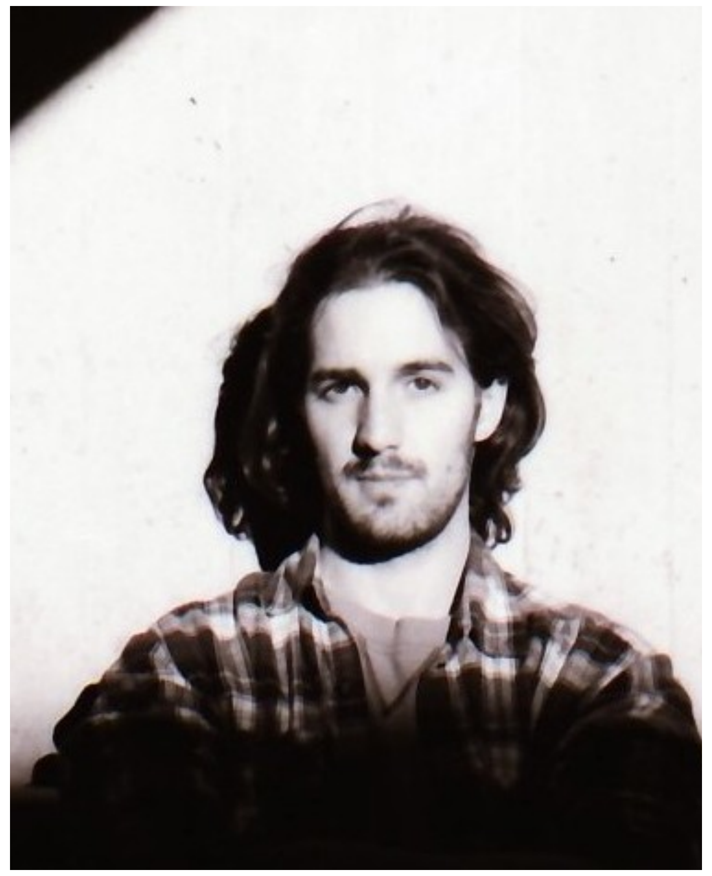
Paper	arXiv:2605.13289

- High-fidelity inference of power grid frequency distributions (In preparation)

Slides, references, and more

Wanru Hao
UK

Jacques Maritz
South Africa



Leonardo Rydin Gorjão
Netherlands / Norway

Christian Beck
UK

Benjamin Schäfer
Germany

Stability of the South African Power Grid: A data-driven statistical physics approach

UK Research and Innovation

Chantelle Van Staden
South Africa

Cristina Trois
South Africa

Thank you!

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