From gridlock to sustainable transport through flow modeling

Alessandro Lonardi

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Social Complexity Lab - DTU

Researcher in **complex systems**

1. Engineering and society
2. Fundamental questions

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- Urban transportation
- Power grids

Researcher in **complex systems**

1. **Engineering and society**
2. **Fundamental questions**

- Urban transportation
- Power grids

- Statistical inference
- Generative models
- Explainable machine learning

BSc Physics
MSc Math. Engineering
PhD Computer Science
Industry Machine Learning
Postdoc Physics + Engineering

2015

Now

Reference

Bilevel Optimization for Traffic Mitigation in Optimal Transport Networks
Alessandro Lonardi, Caterina De Bacco
Phys. Rev. Lett. 2023 arXiv GitHub

Let’s talk **engineering and society: urban transportation…**

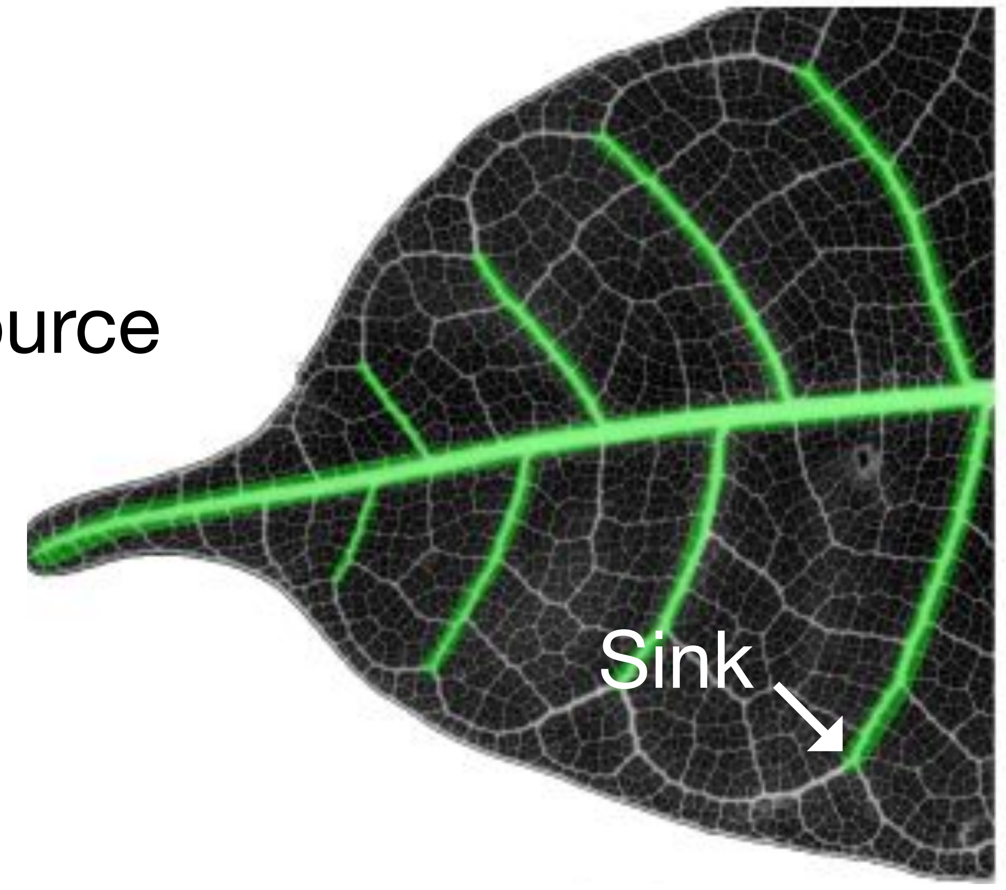
Let’s talk **engineering and society: urban transportation…**

…with a paradigm shift

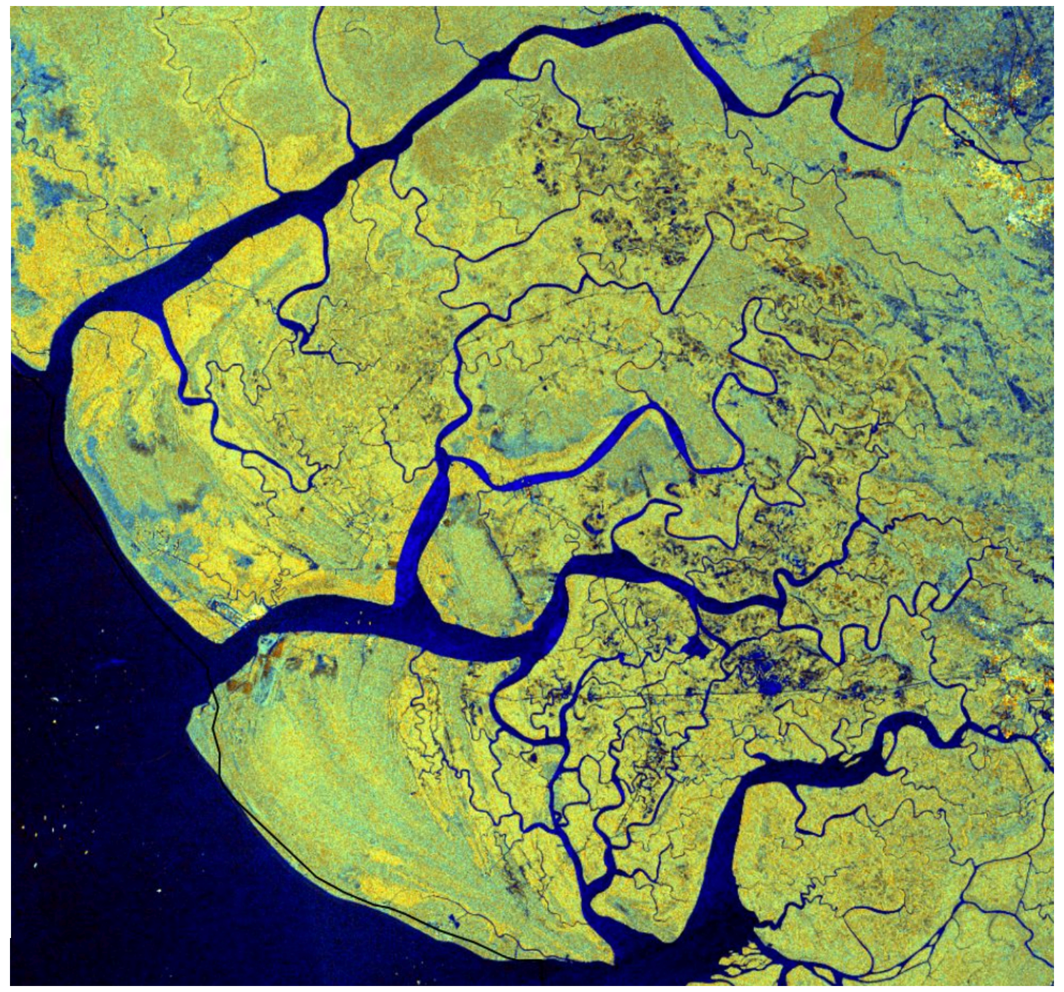
Transport networks are pervasive at all scales

Natural systems

Source

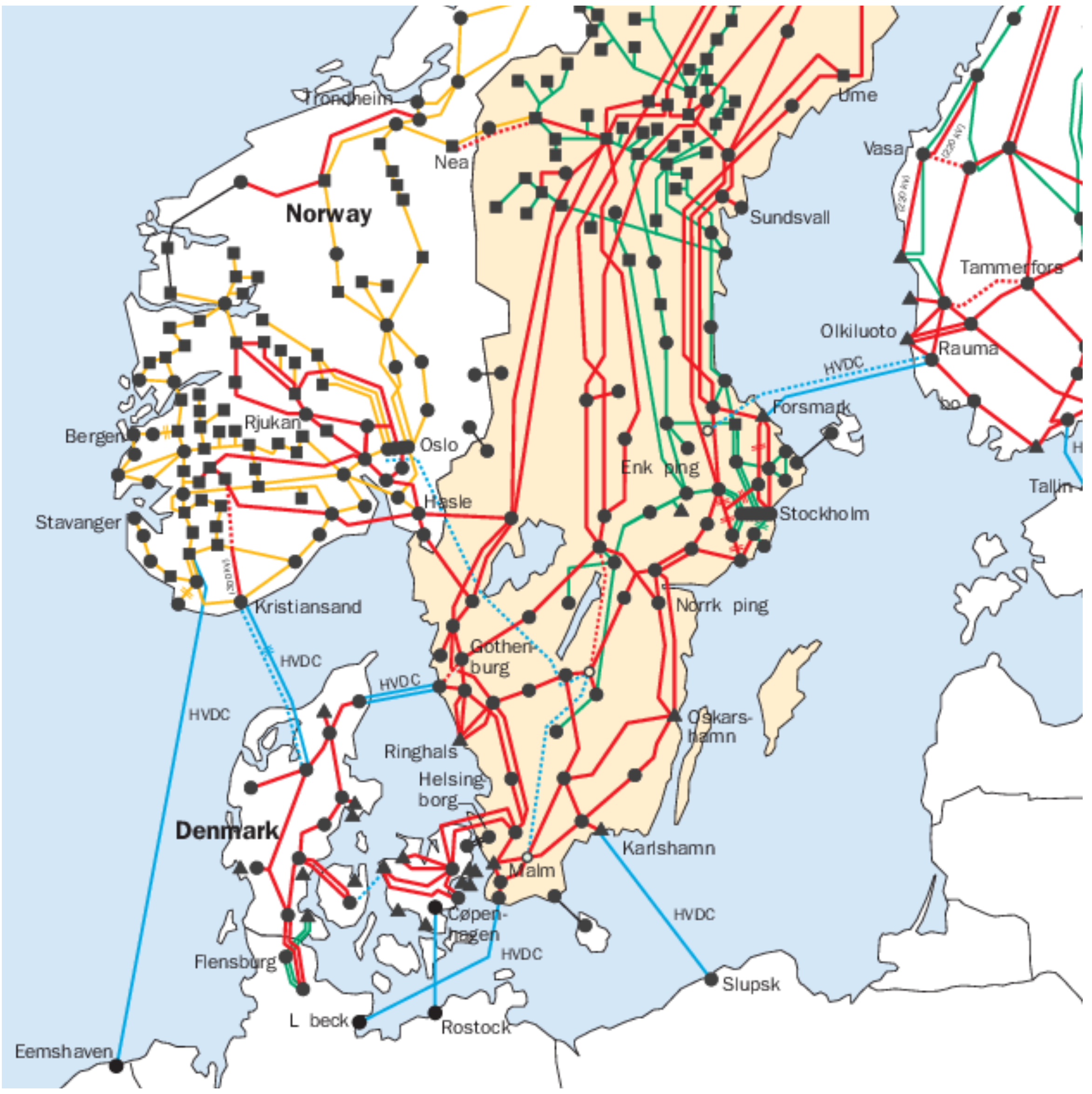


Ronellenfitsch, Katifori
Phys. Rev. Lett. 2016



European Space Agency

Artificial systems



Perninge
KTH 2011

Transport for
London

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Natural systems

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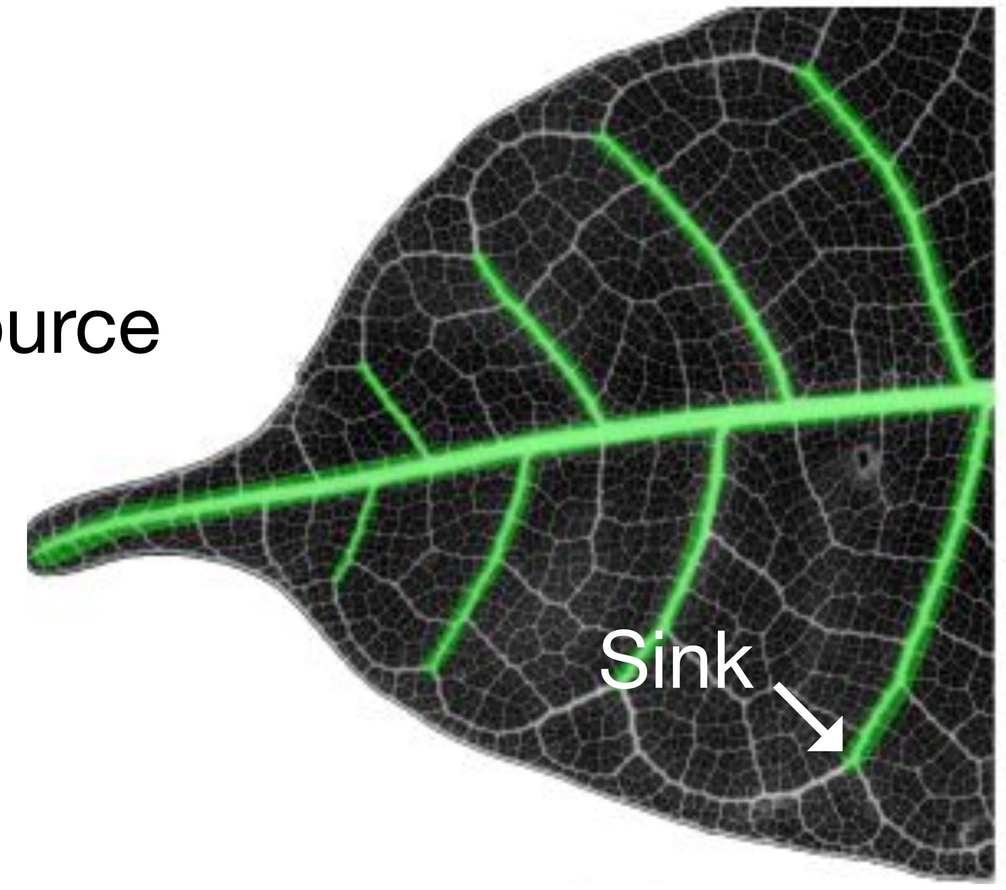

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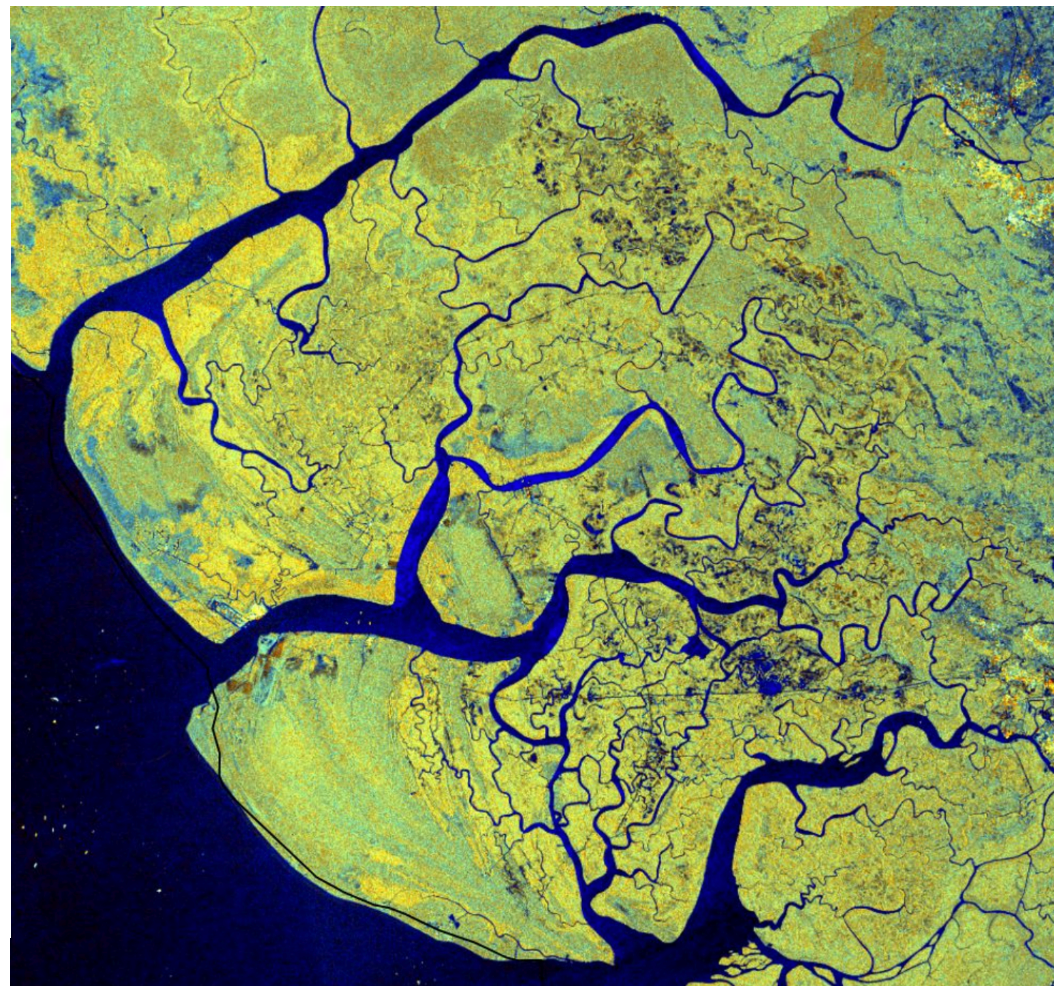

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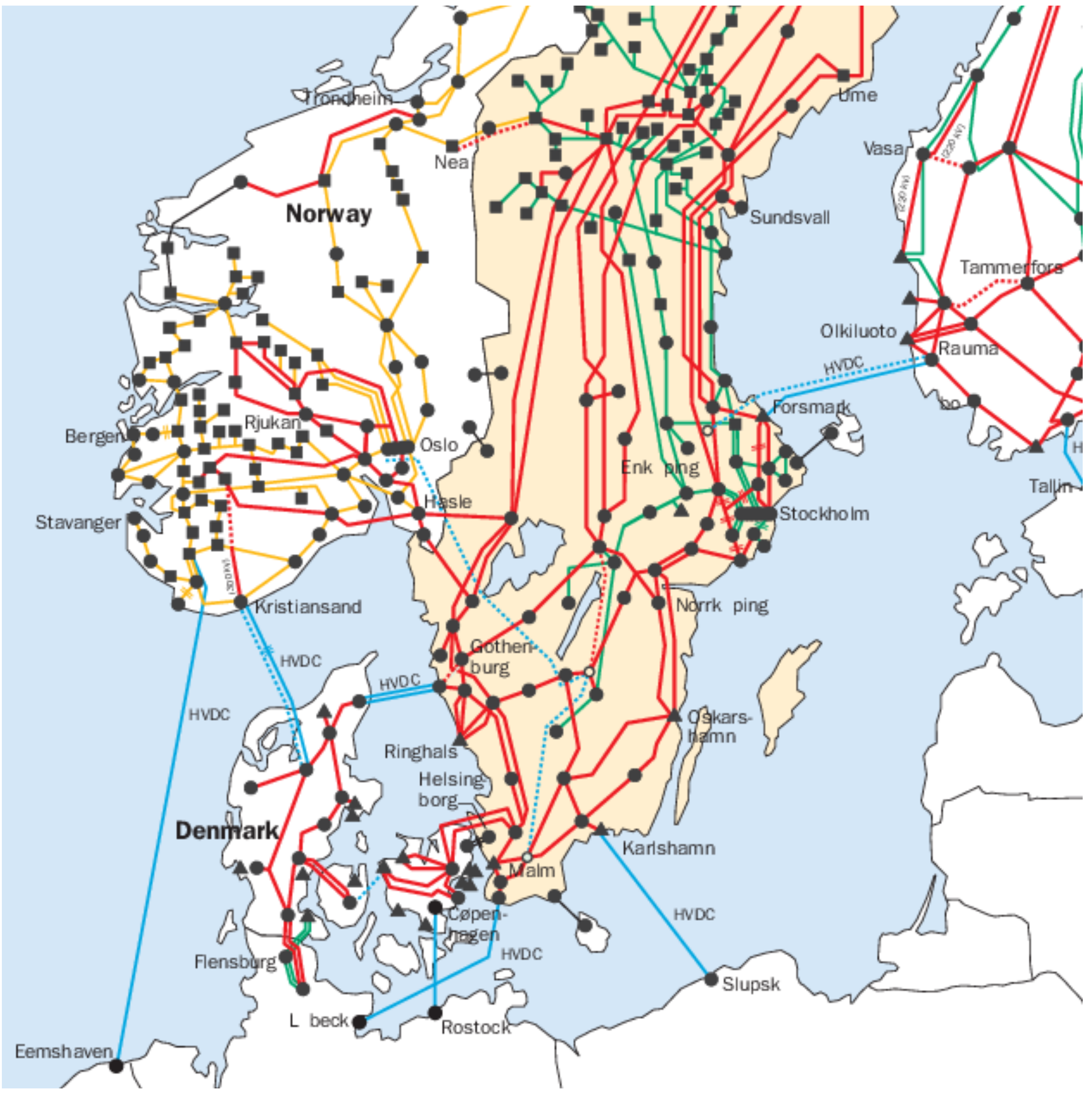
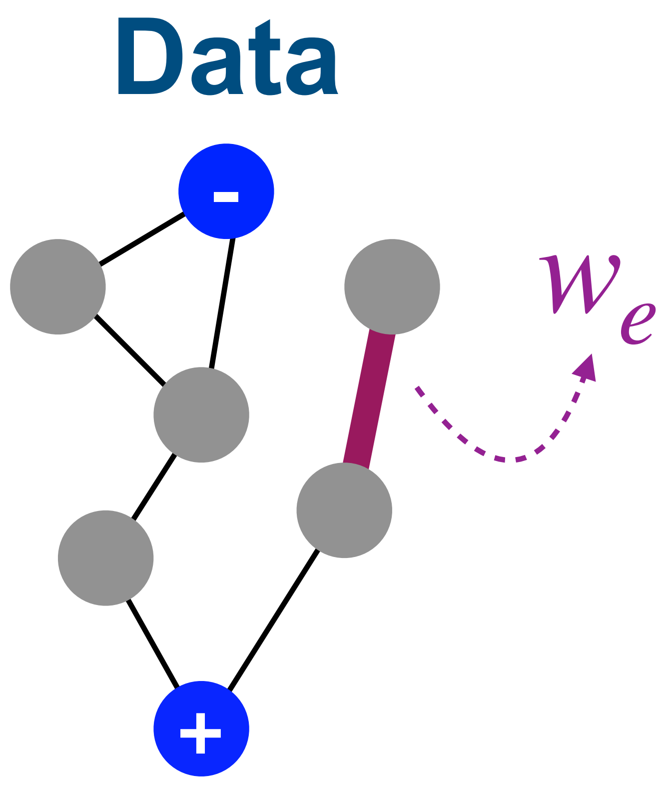


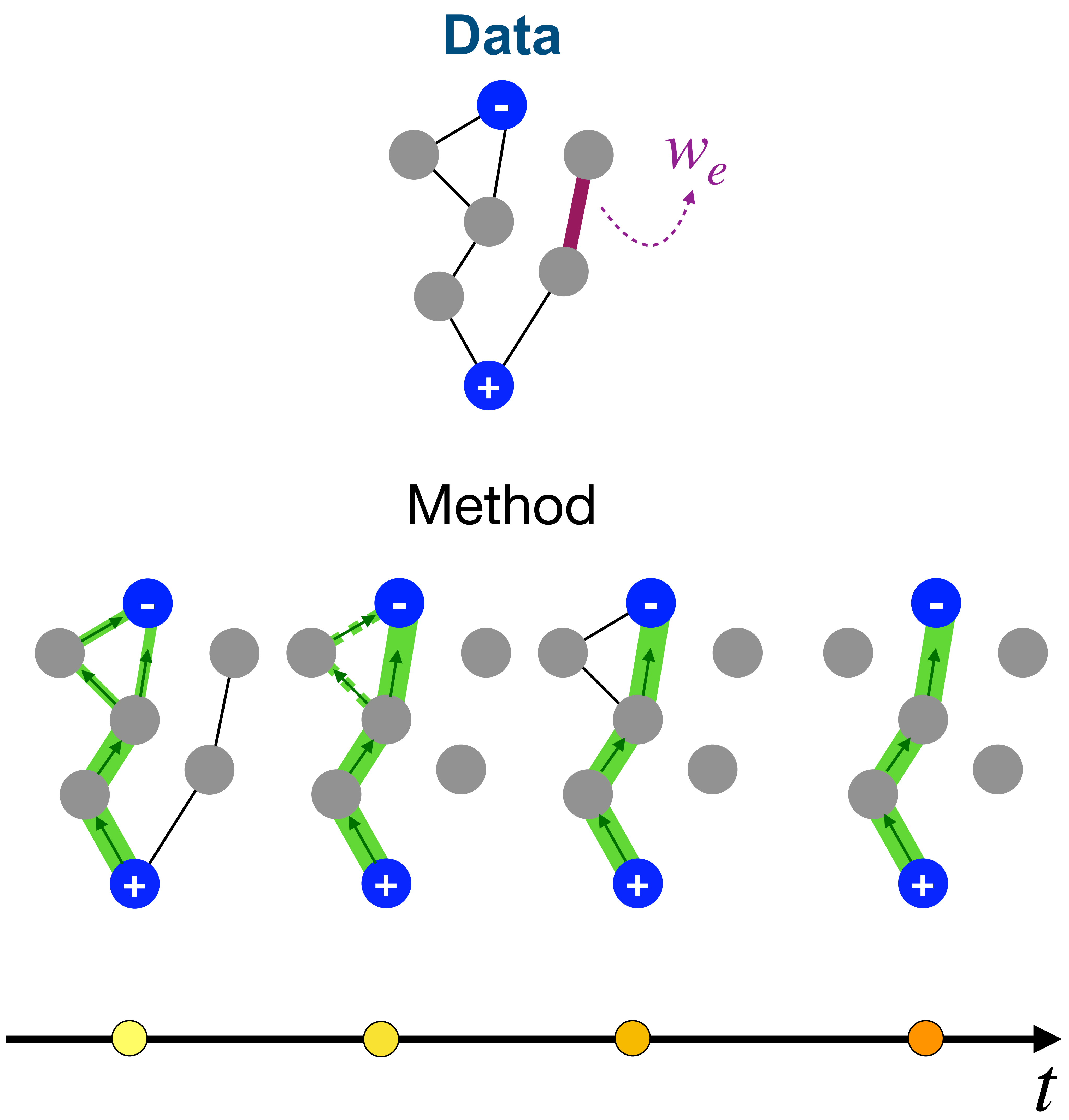
Perninge
KTH 2011

Transport for
London

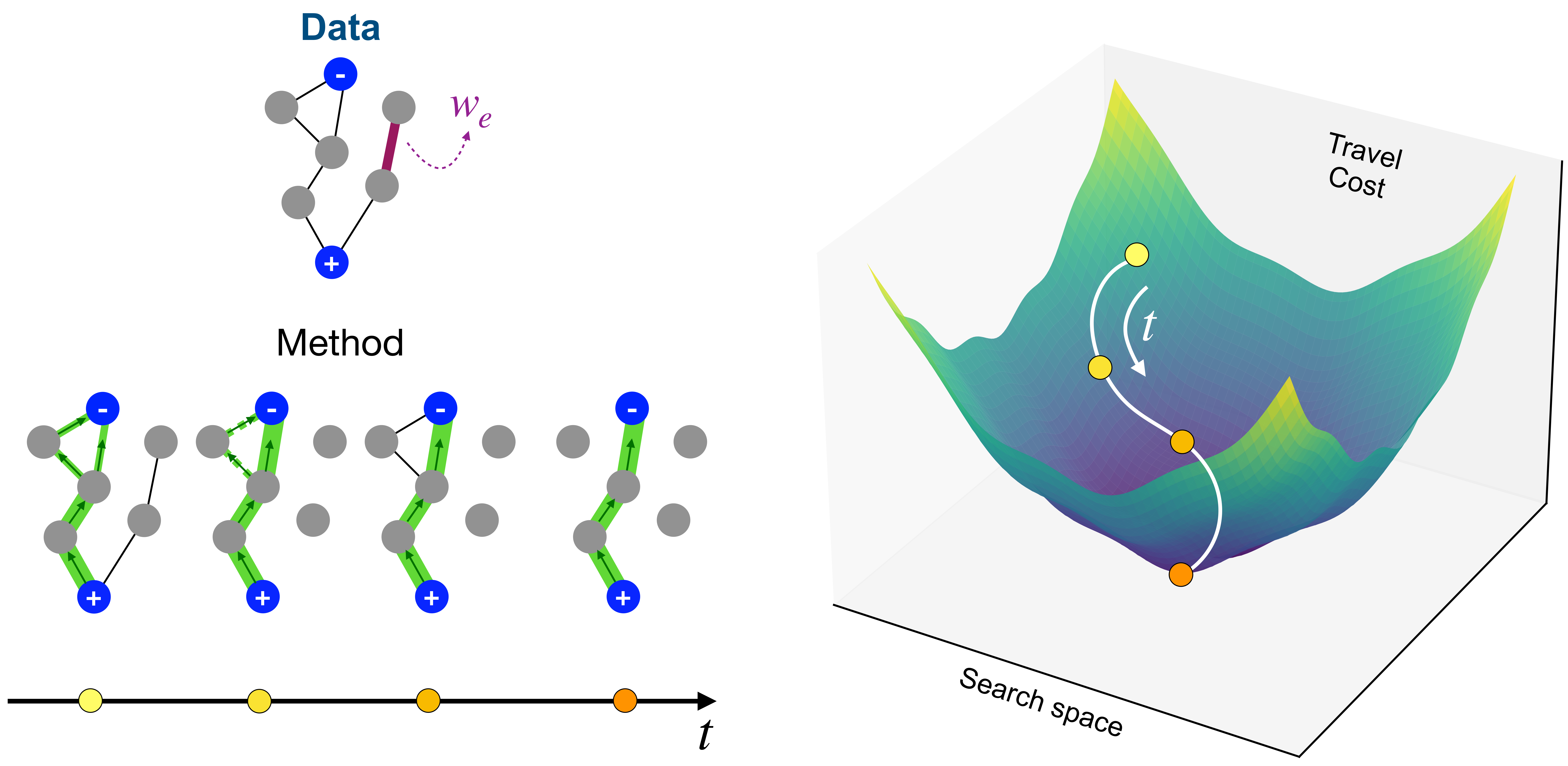
Evidence: Adaptation leads to the emergence of transport networks

Broader goal: Leverage adaptation principles to devise **principled optimization methods** and **algorithms** for network design tasks

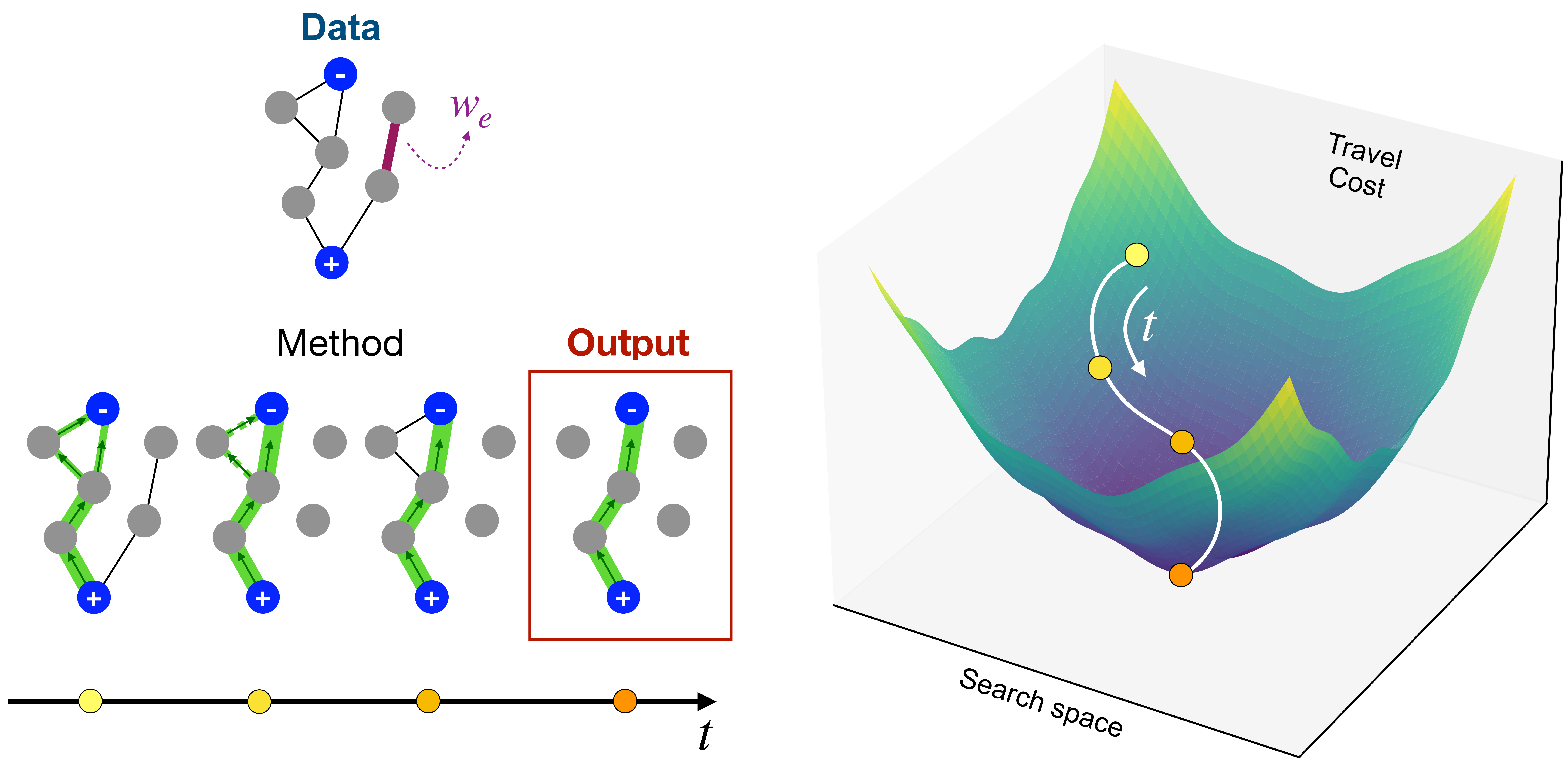

Data
-
+
w_e



Data
w_e
Method
-
+
t

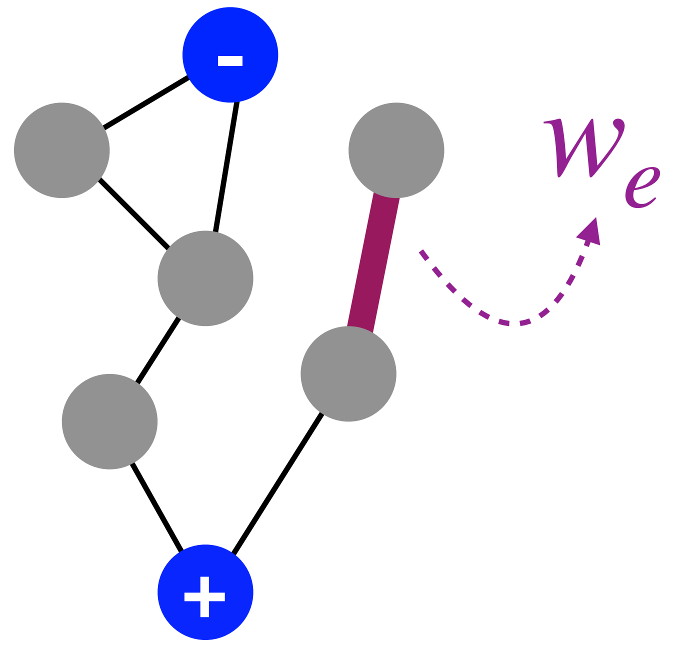


Data
w_e
Method
t
Travel Cost
t
Search space



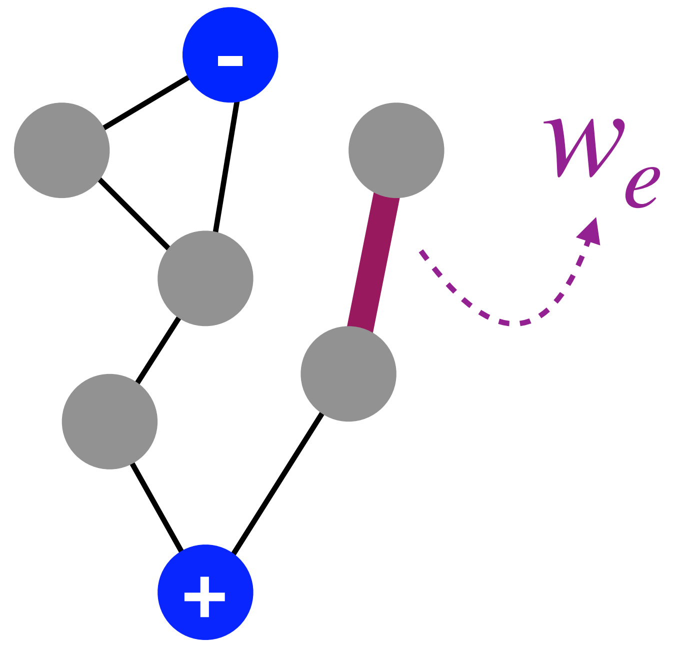
Data
w_e
Method
Output
t
Travel Cost
t
Search space

Questions?



μ_e : edge capacity

F_e : edge flux



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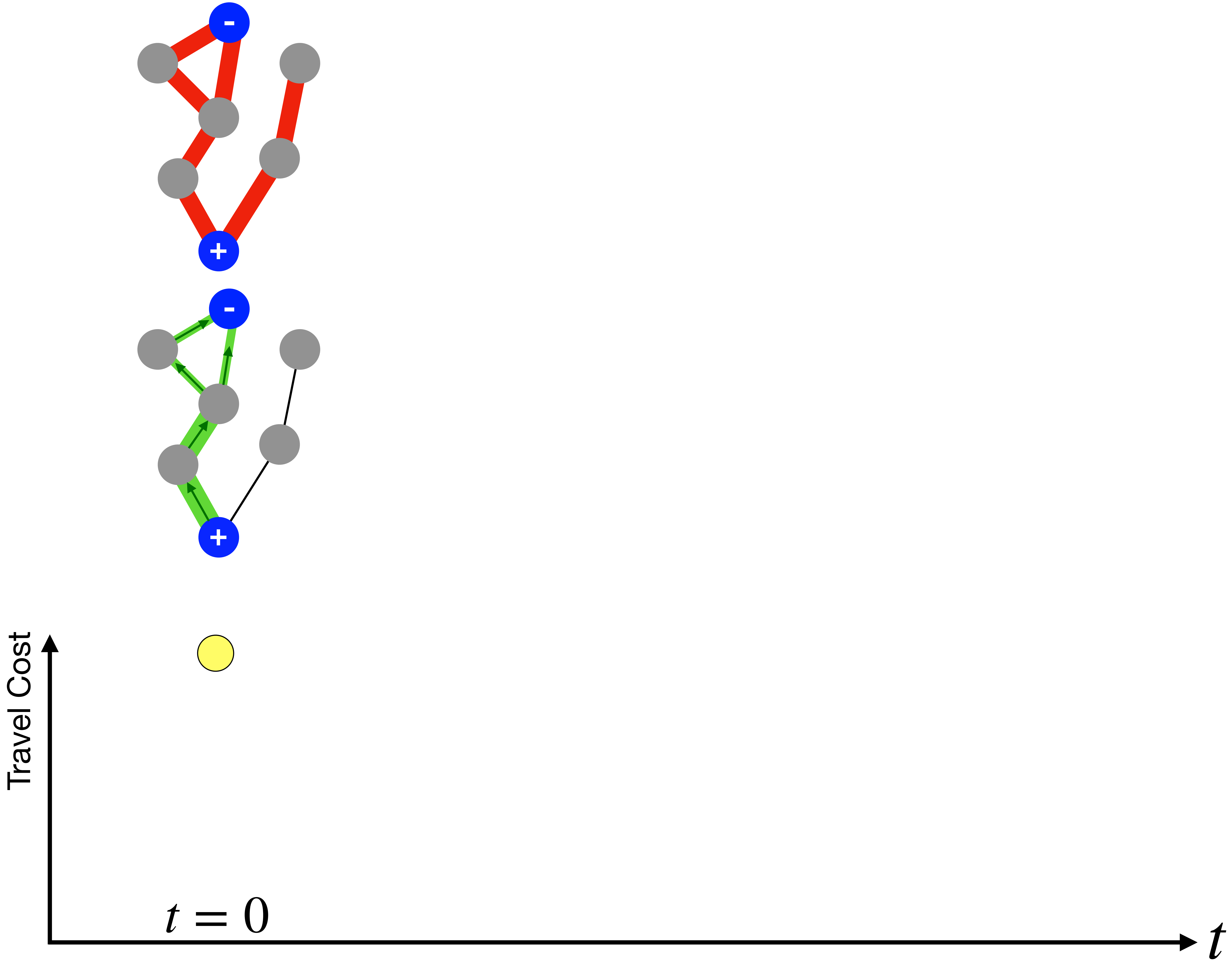
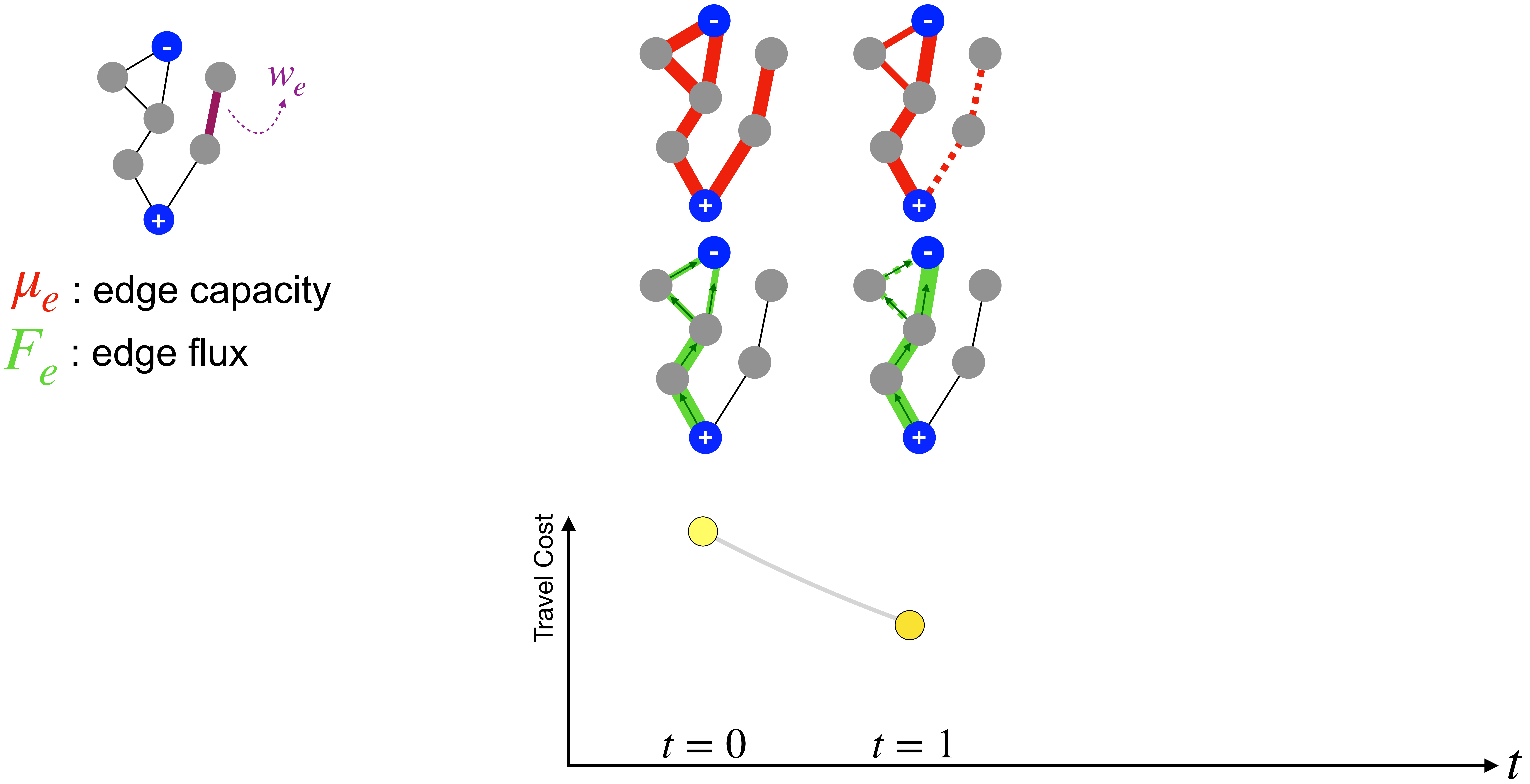
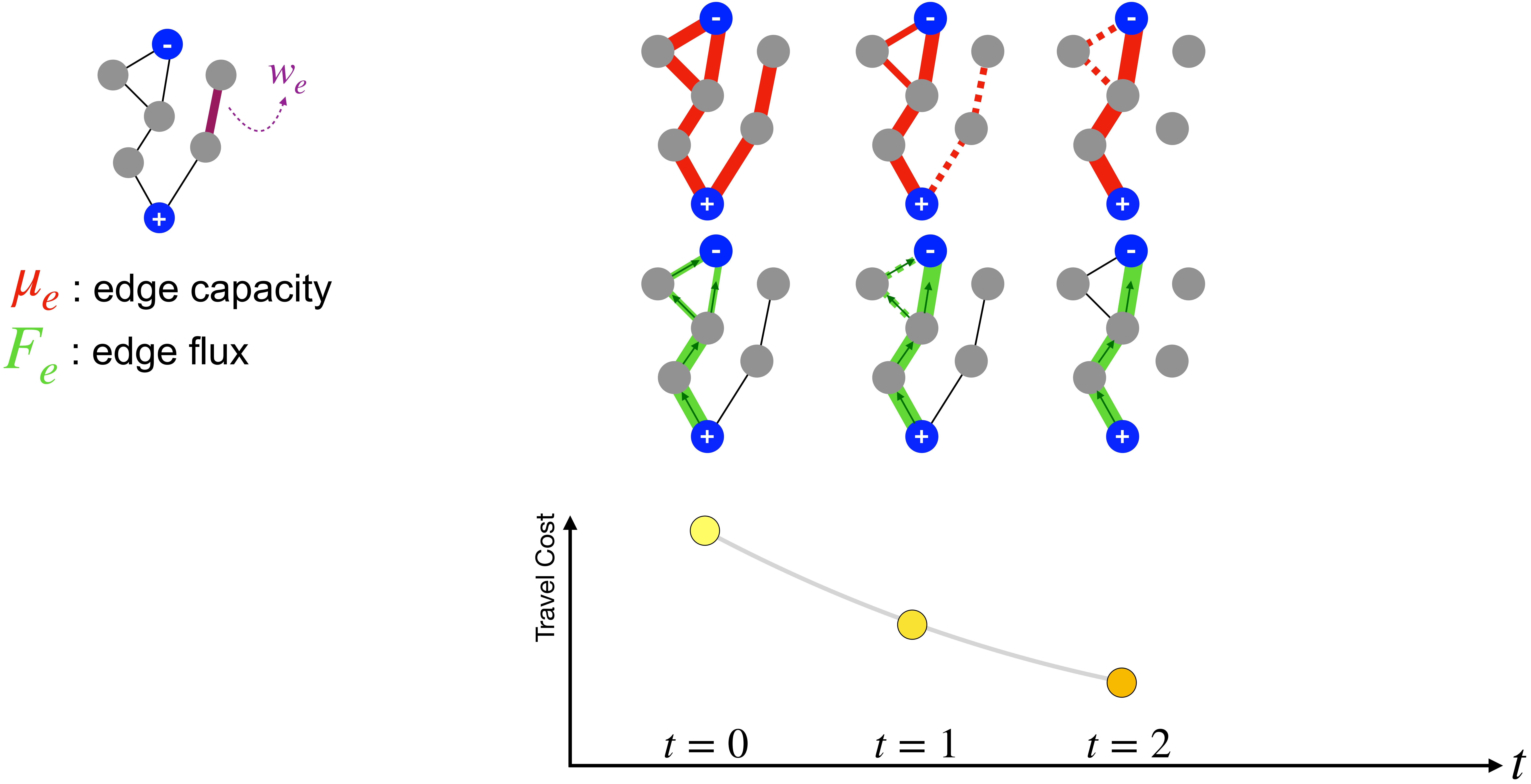
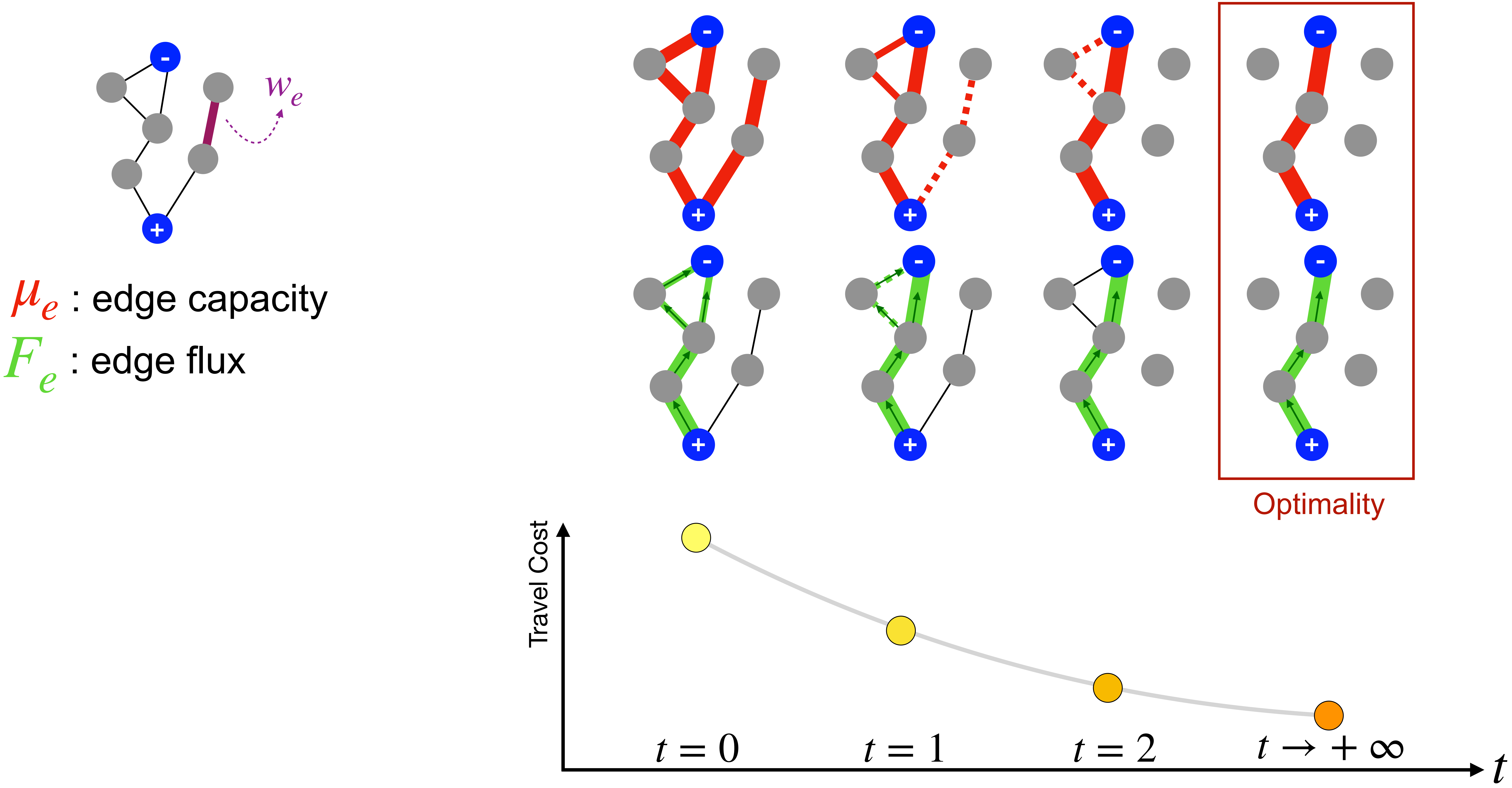




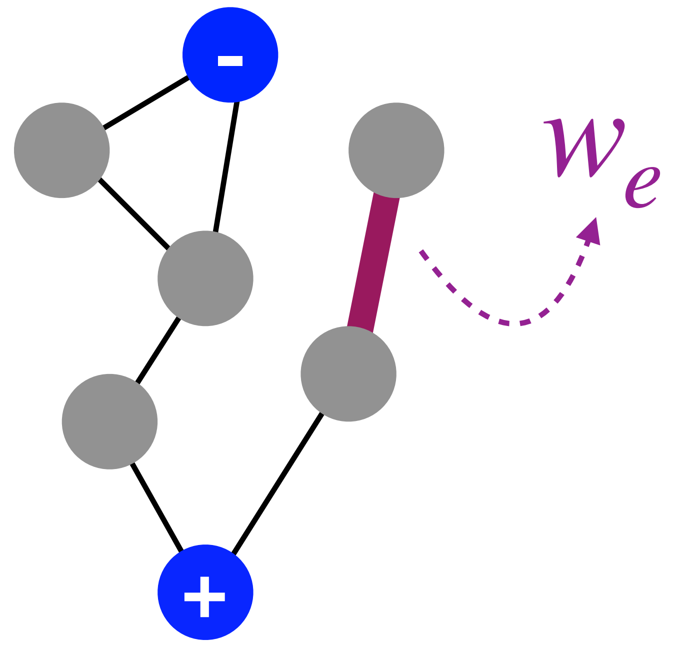
w_e
μ_e : edge capacity
F_e : edge flux
Travel Cost
$t = 0$
$t = 1$
t



w_e
μ_e : edge capacity
F_e : edge flux
Travel Cost
$t = 0$
$t = 1$
$t = 2$
t



w_e
μ_e : edge capacity
F_e : edge flux
Optimality
Travel Cost
$t = 0$
$t = 1$
$t = 2$
$t \to +\infty$
t

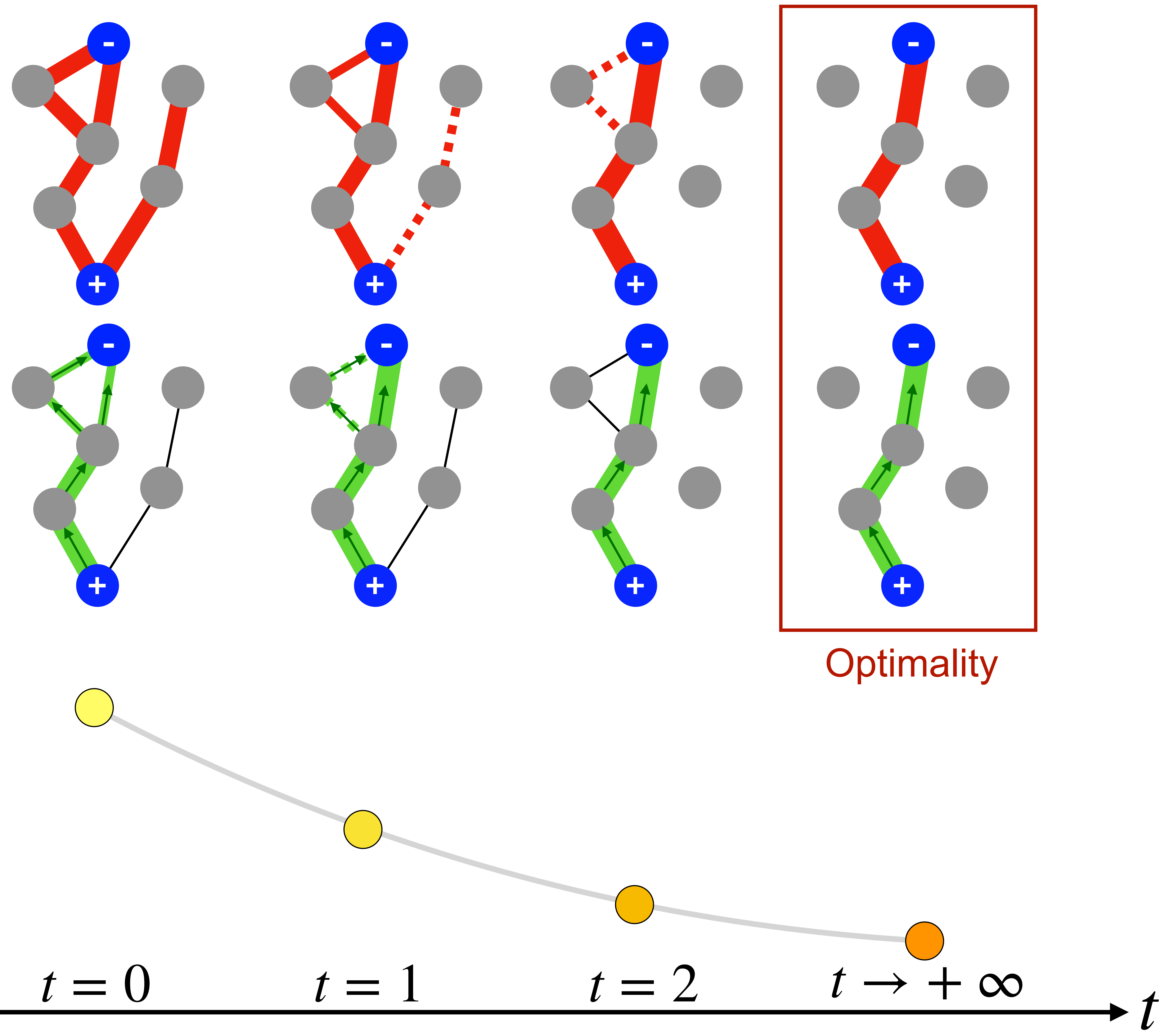


μ_e : edge capacity

F_e : edge flux

$e = (u, v)$

$$
\begin{cases}
\dfrac{d\mu_e}{dt} = |F_e| - \mu_e \\
F_e(\mu, p) = \mu_e(p_u - p_v)/w_e \\
\text{Conservation of mass}
\end{cases}
$$



- The dynamics can compute **shortest paths**: **Lyapunov functional** [Bonifaci *et al.* J. Theor. Bio. 2012]

- The minimum of the Lyapunov functional is the **Wasserstein distance** between entry and exit mass [Facca *et al.* SIAM J. Appl. Math 2018, Facca *et al.* J. Sci. Com. 2020]

- **Computational advancements**: dynamical systems' solvers [Facca, Benzi. J. Sci. Com. 2021], connection with mirror gradient descent [Bonifaci Comput. Optim. Appl. 2021]

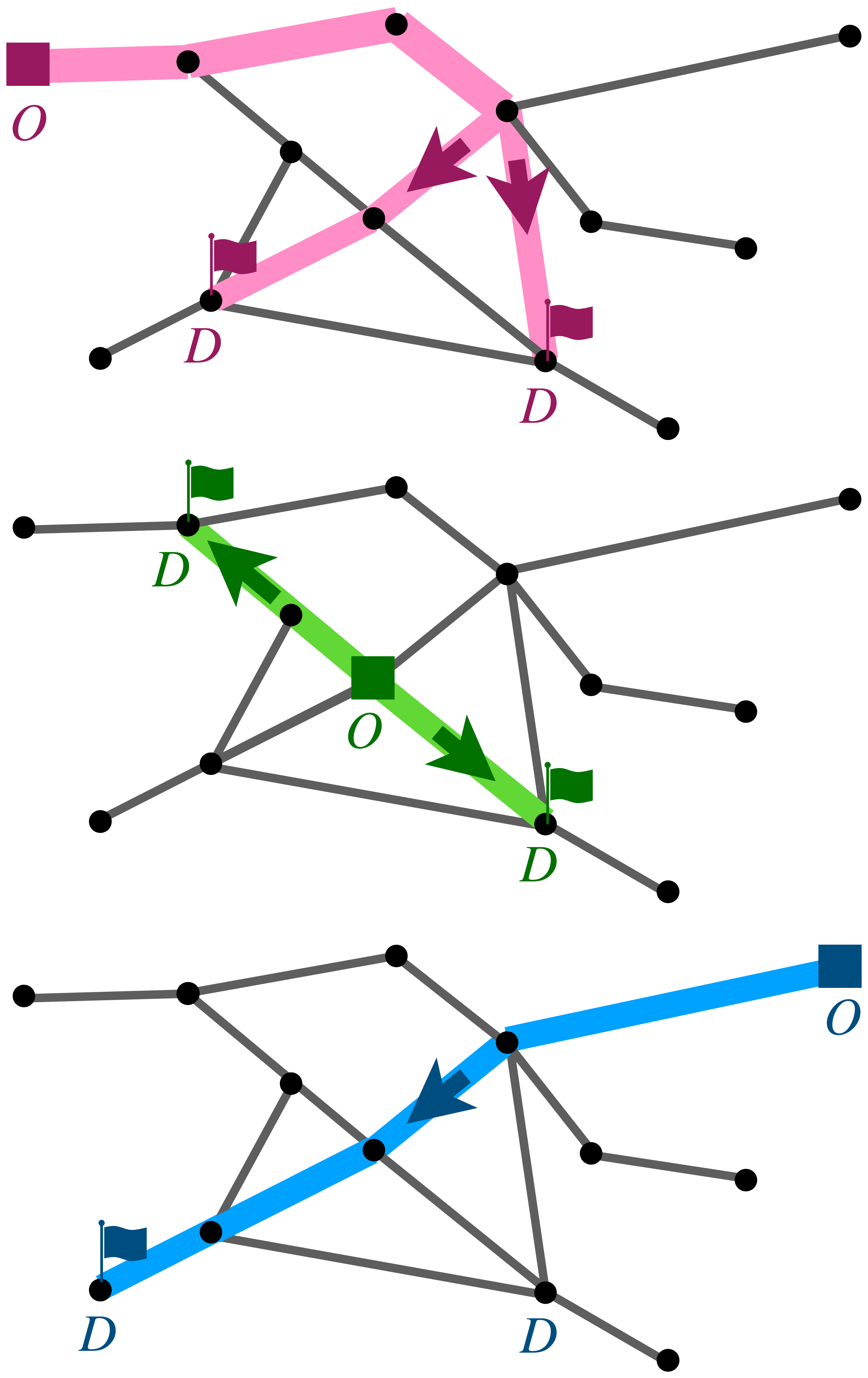
Takeaway

Dynamical systems can be an efficient tool to solve optimization problems

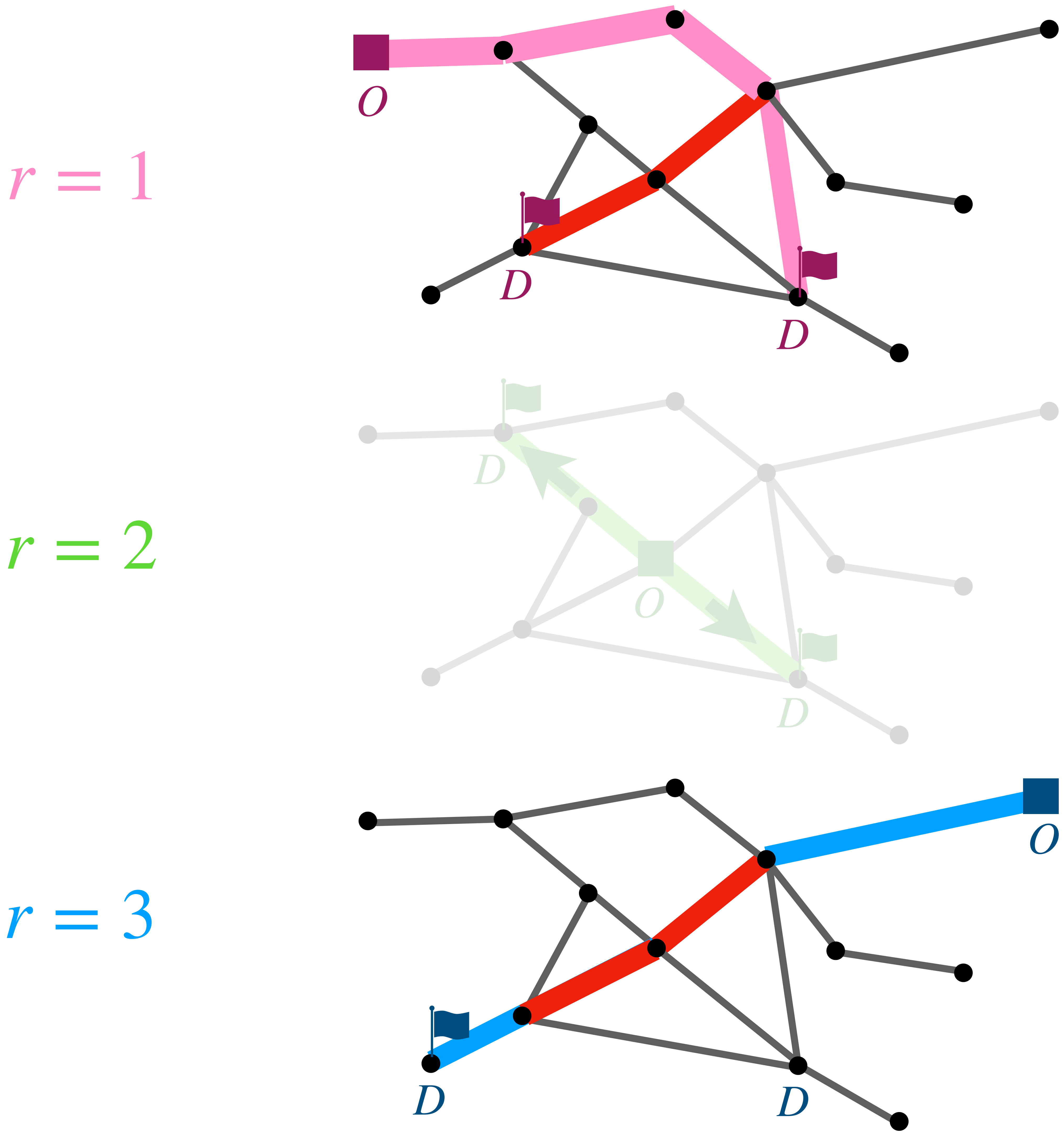
$$\frac{d\mu_e}{dt} = |F_e| - \mu_e \longleftrightarrow \hat{\mu} = \mathrm{argmin}_\mu \text{Travel Cost}(\mu;\, w)$$

Questions?

nlc.org



- Passengers travel greedily from one **O**rigin to multiple **D**estinations (Inflows and Outflows)

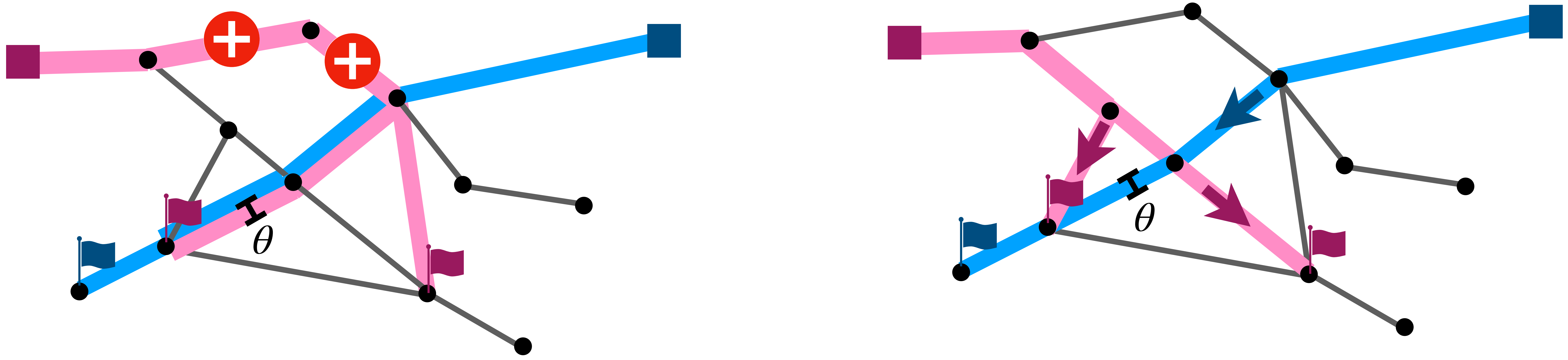


- Passengers travel greedily from one **O**rigin to multiple **D**estinations (Inflows and Outflows)

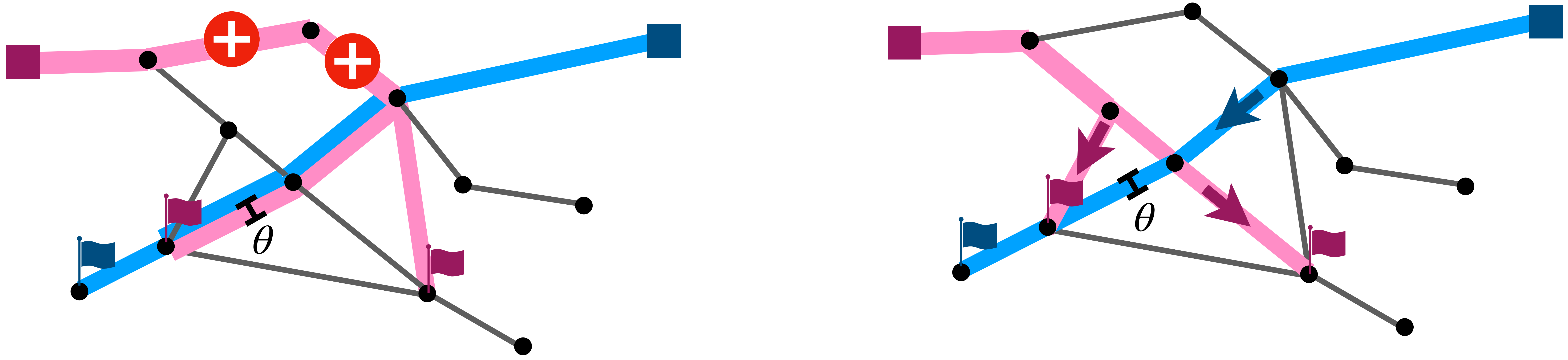
- Passengers trigger **traffic congestion**

$$\sum_r |F_e^r| \geq \theta$$

A network manager tunes the edge weights to mitigate traffic



A network manager tunes the edge weights to mitigate traffic



We pose a bilevel optimization problem

$$\min_{w} \text{Congestion Cost}_{\theta}(w\,;\hat{\mu}) \,:\, \hat{\mu} = \operatorname{argmin}_{\mu} \text{Travel Cost}(\mu; w)$$

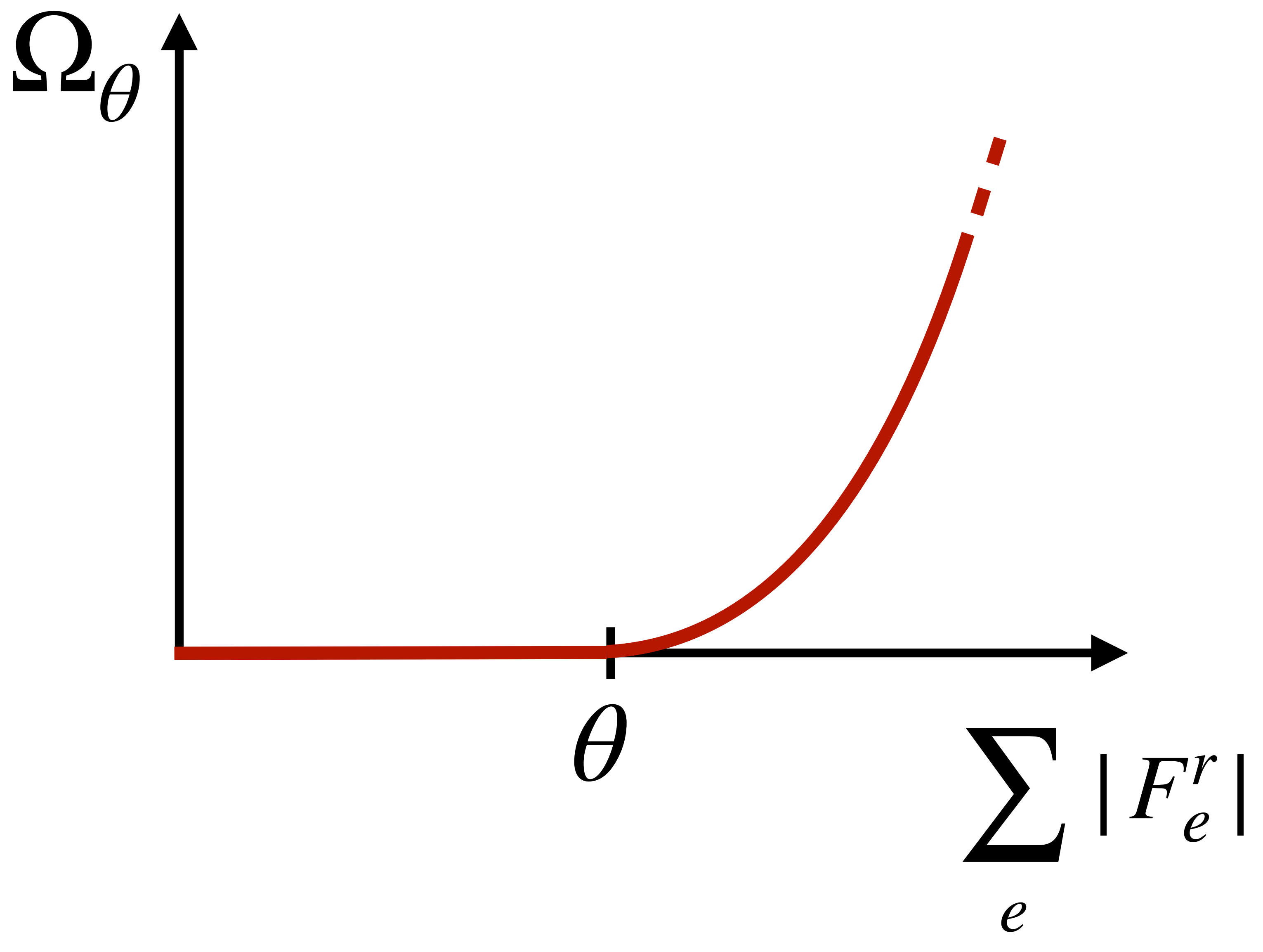
$$\hat{\mu} = \mathrm{argmin}_{\mu} \text{Travel Cost}(\mu; w)$$

$$J = \sum_{e,r} w_e \, | F_e^r(\mu, w) |$$

$$\min_{w} \text{Congestion Cost}_{\theta}(w; \hat{\mu}) : \hat{\mu} = \text{argmin}_{\mu} \text{Travel Cost}(\mu; w)$$

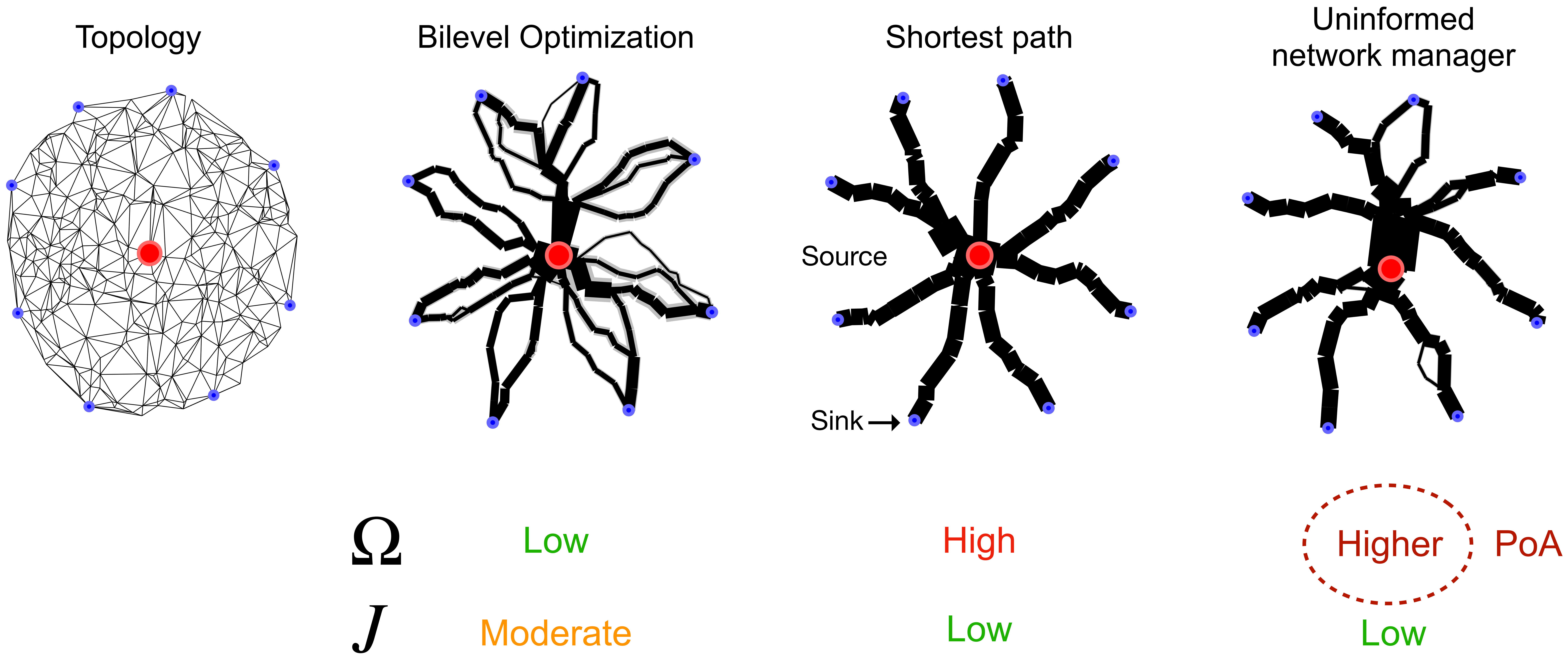
$$\Delta_e := \sum_r |F_e^r| - \theta \ : \ \Omega_\theta = \sum_e \Delta_e^2 H(\Delta_e)$$

$$J = \sum_{e,r} w_e |F_e^r(\mu, w)|$$



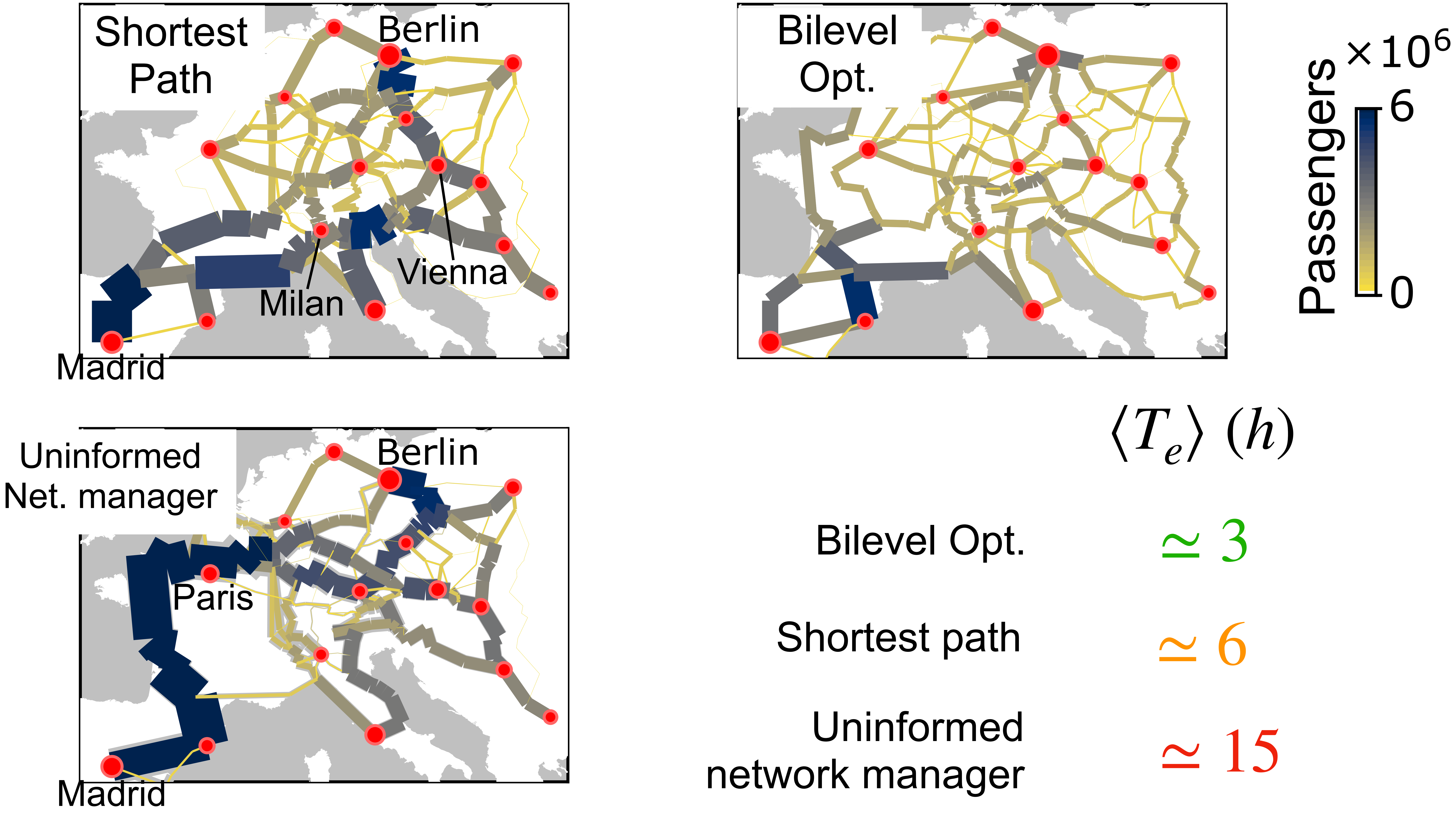
Closed-form equations

$$
\begin{cases}
\dfrac{d\mu_e^r}{dt} = |F_e^r| - \mu_e^r \\
w \leftarrow \mathrm{PGSD}[\Omega_\theta(w\,;\mu)] \\
F_e^r(\mu, p) = \mu_e(p_u - p_v)/w_e \\
\text{Conservation of mass}
\end{cases}
$$



Topology
Bilevel Optimization
Shortest path
Uninformed network manager
Source
Sink
Ω
Low
High
Higher
PoA
J
Moderate
Low
Low

International European Highways



	$\langle T_e \rangle$ (h)
Bilevel Opt.	$\simeq 3$
Shortest path	$\simeq 6$
Uninformed network manager	$\simeq 15$

Takeaway

Dynamical systems and toll tuning can inform us on the nature of traffic congestion

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Dynamical systems can be an efficient tool to solve optimization problems

Dynamical systems and toll tuning can inform us on the nature of traffic congestion

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Alessandro Lonardi, Caterina De Bacco
Phys. Rev. Lett. 2023 arXiv GitHub

Thank you!

Let's connect, even — ***especially*** — if our expertise doesn't overlap!

📧 alessandro.lonardi.vr@gmail.com
🌐 aleable.github.io

💻 github/aleable
🦋 bsky/aleable.bsky.social
💼 linkedin/alonardi