Bilevel Optimization for Traffic Mitigation in Optimal Transport Networks*

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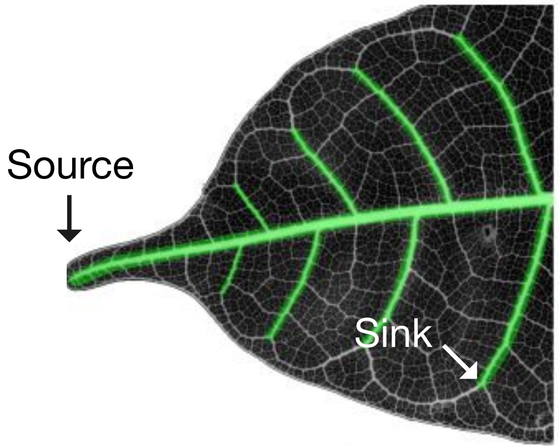
* Bilevel Optimization for Traffic Mitigation in Optimal Transport Networks

Alessandro Lonardi and Caterina De Bacco
Phys. Rev. Lett. **131**, 267401 – Published 26 December 2023

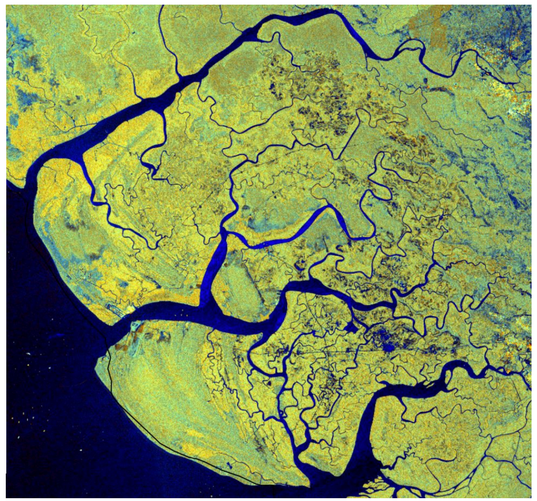
Goal: network design with a paradigm shift

Transport networks are pervasive at all scales

Natural systems

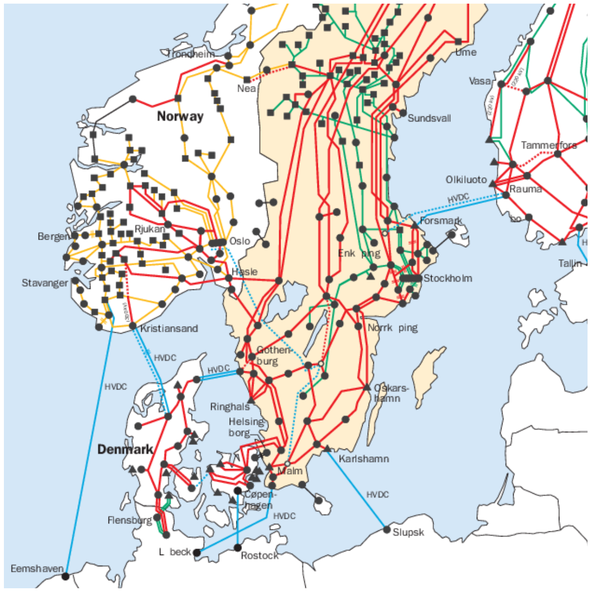


Ronellenfitsch and Katifori
Phys. Rev. Lett. 2016



European Space Agency

Artificial systems



Perninge
KTH 2011

Transport for
London

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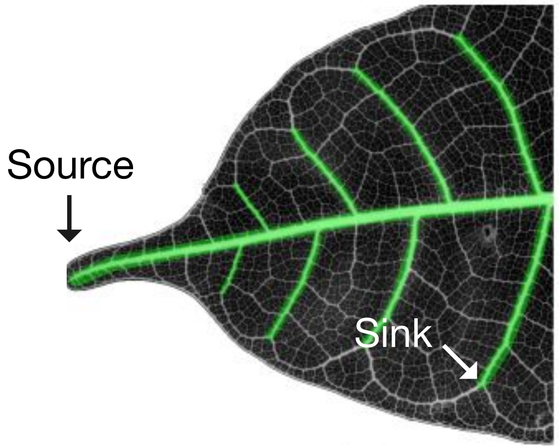

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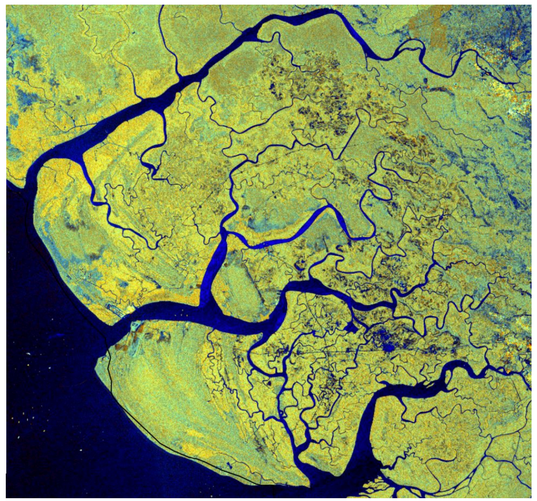

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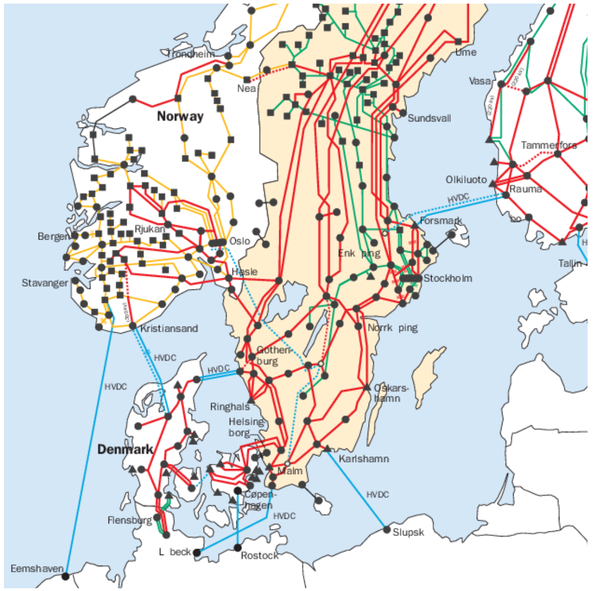


Perninge
KTH 2011

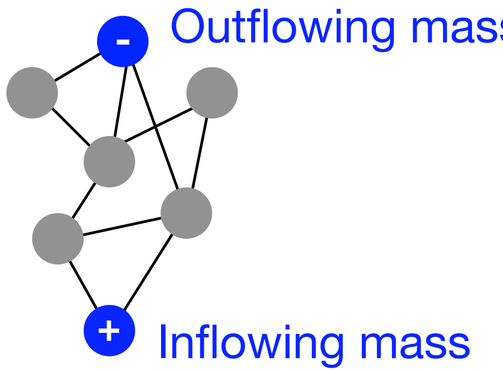
Transport for
London

Evidence: Adaptation leads to the emergence of transport networks

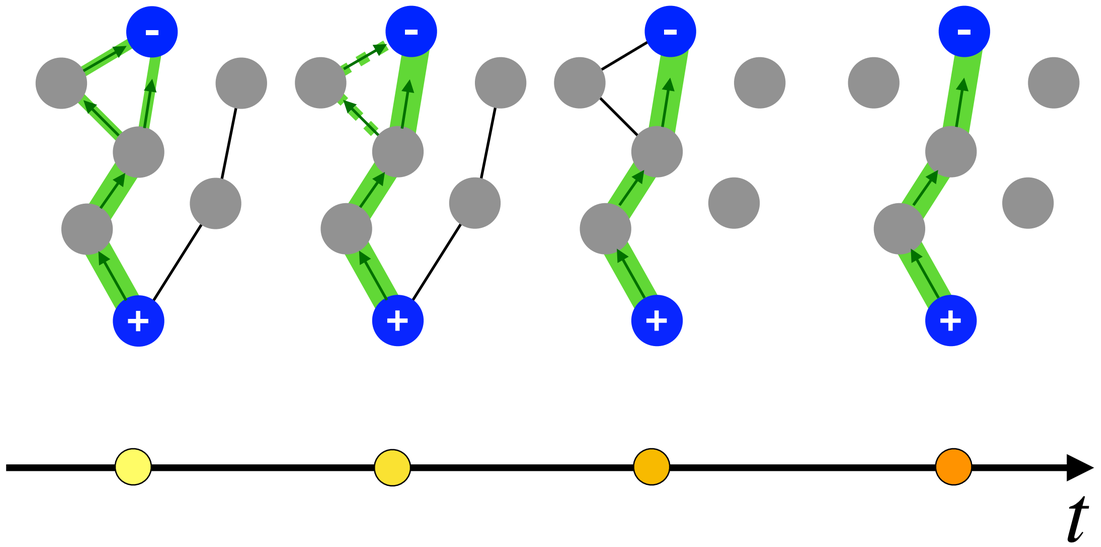
Goal: Leverage adaptation principles to devise **principled optimization methods** and **scalable algorithms** for network design tasks

Background: adaptive networks

Question:

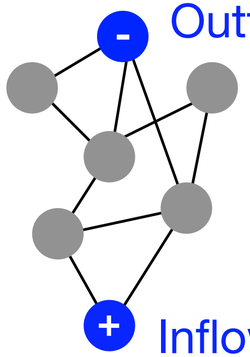


Answer:



Background: optimal Transport

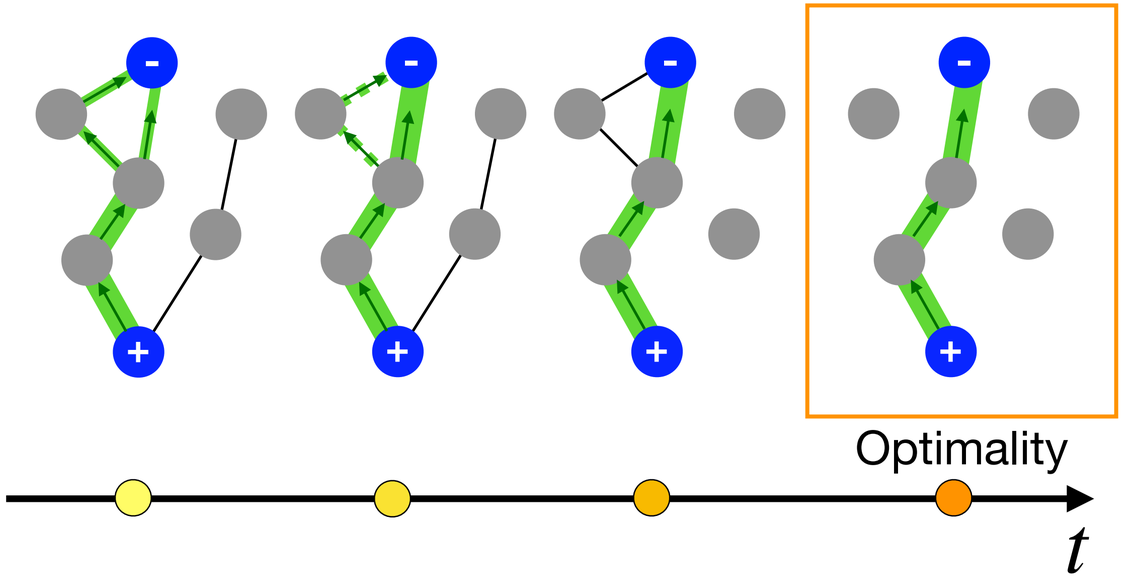
Question:

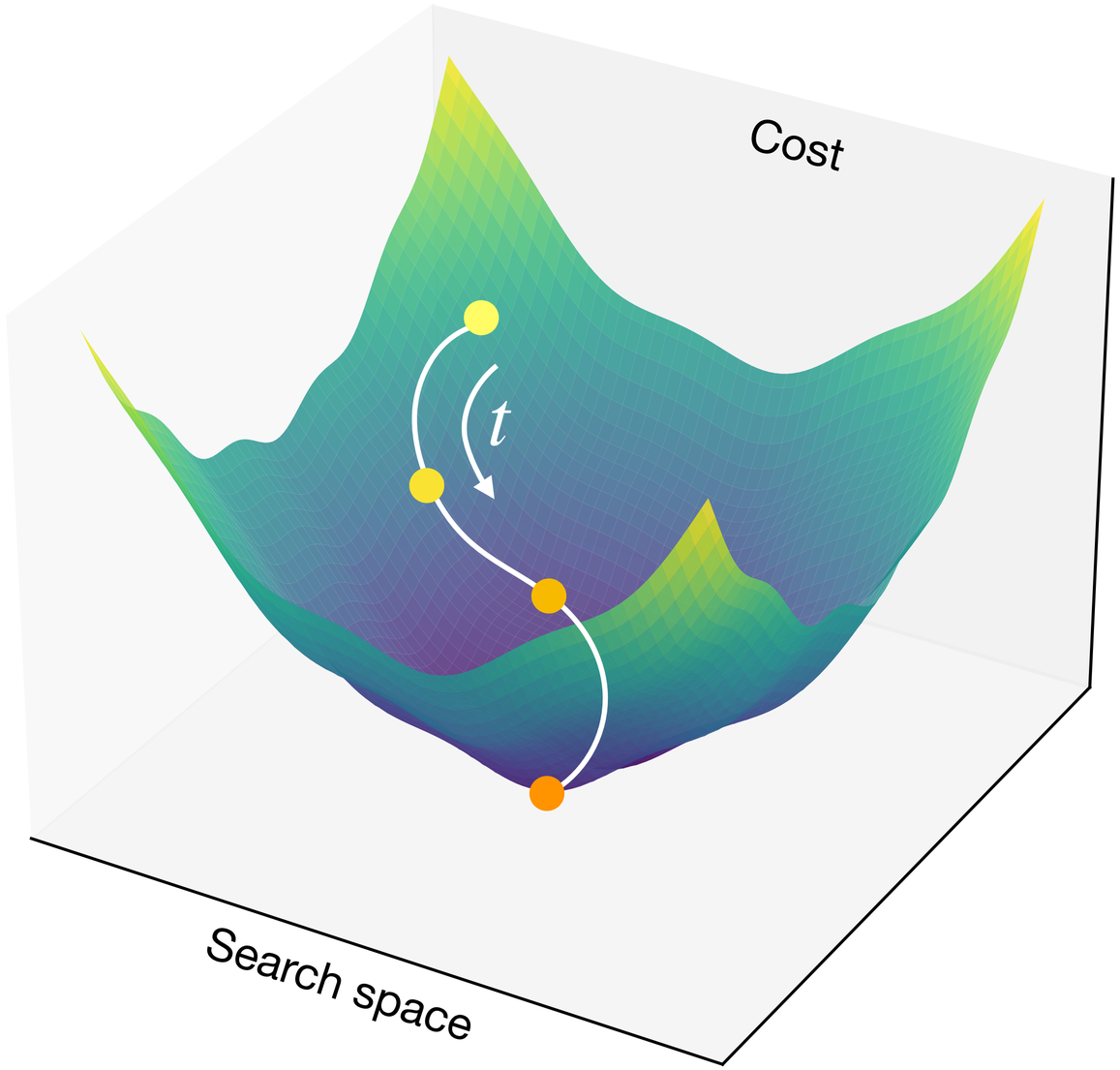


Outflowing mass

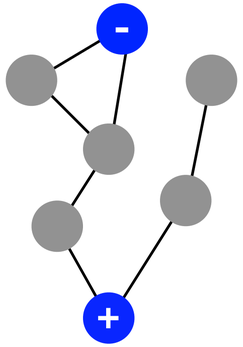
Inflowing mass

Answer:



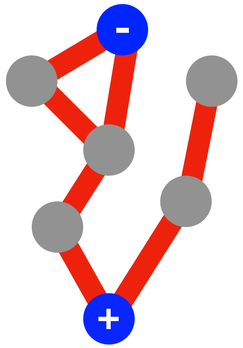


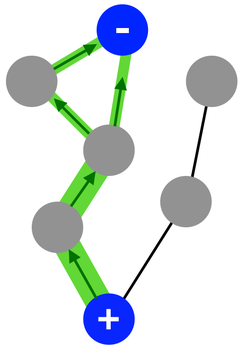
Background: unicommodity routing



μ_e : edge capacity

F_e : flux





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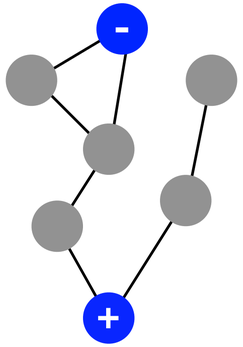

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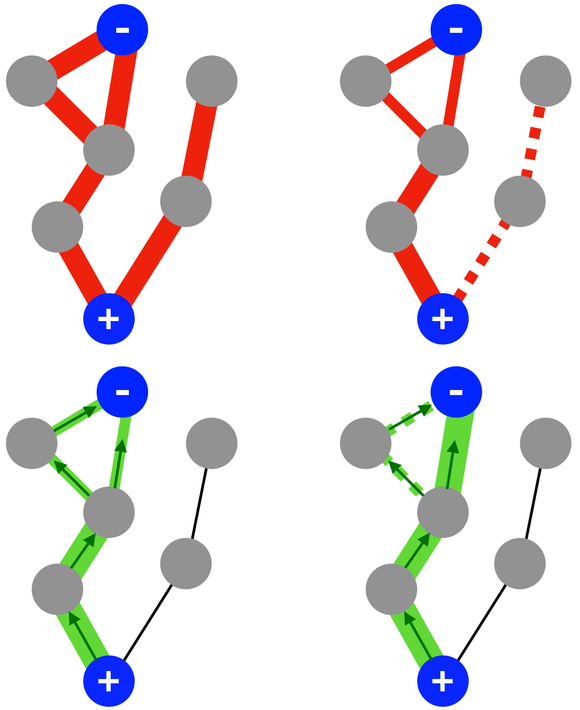

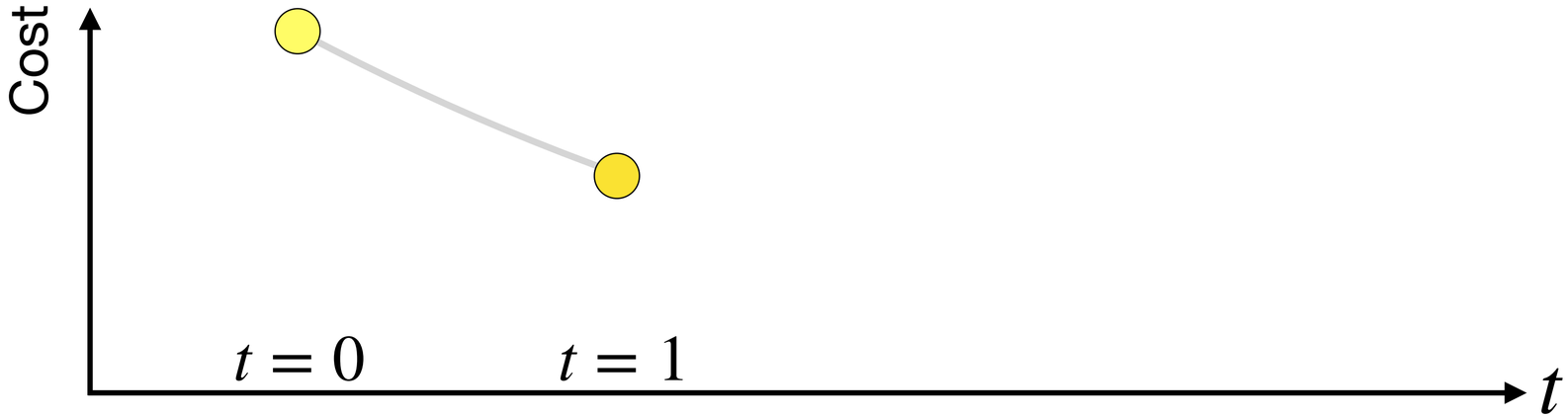

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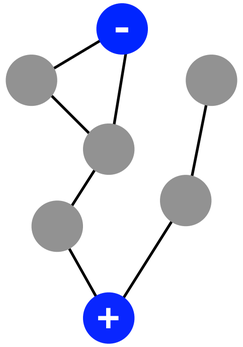

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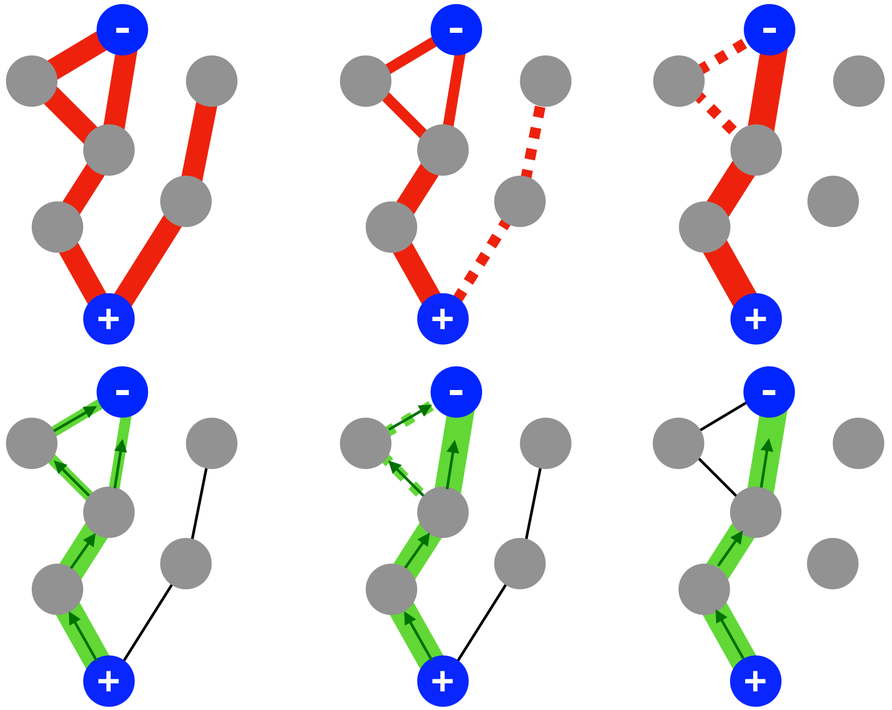

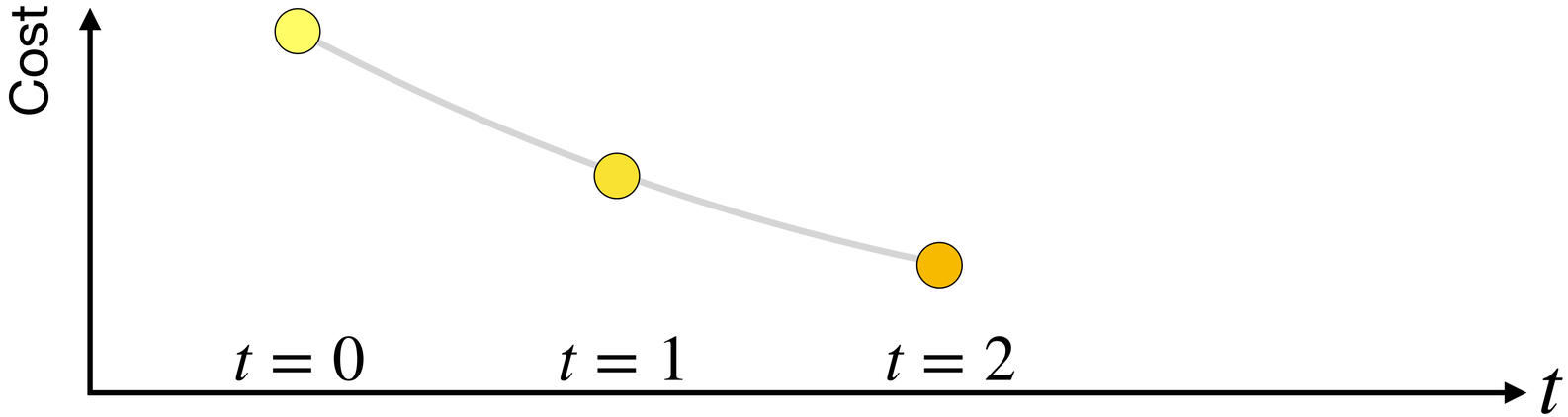

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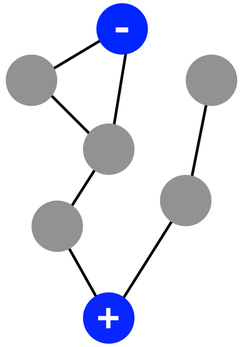

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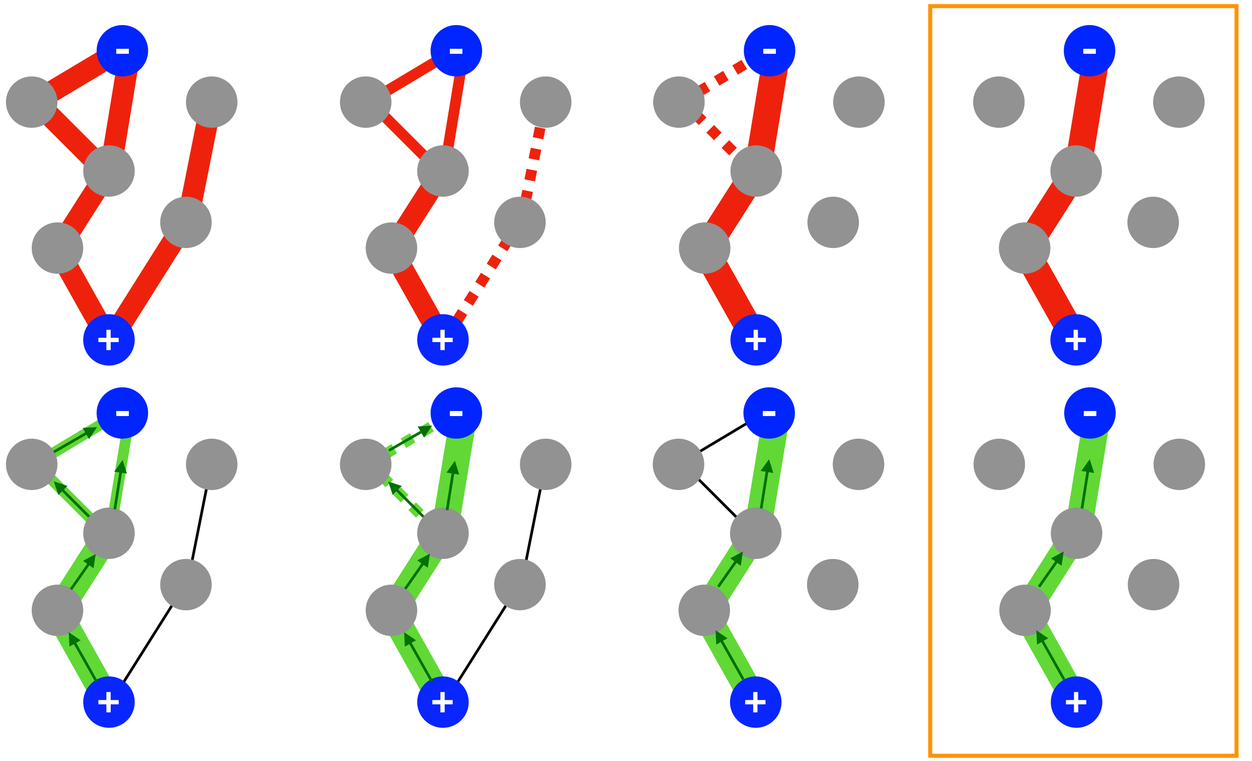
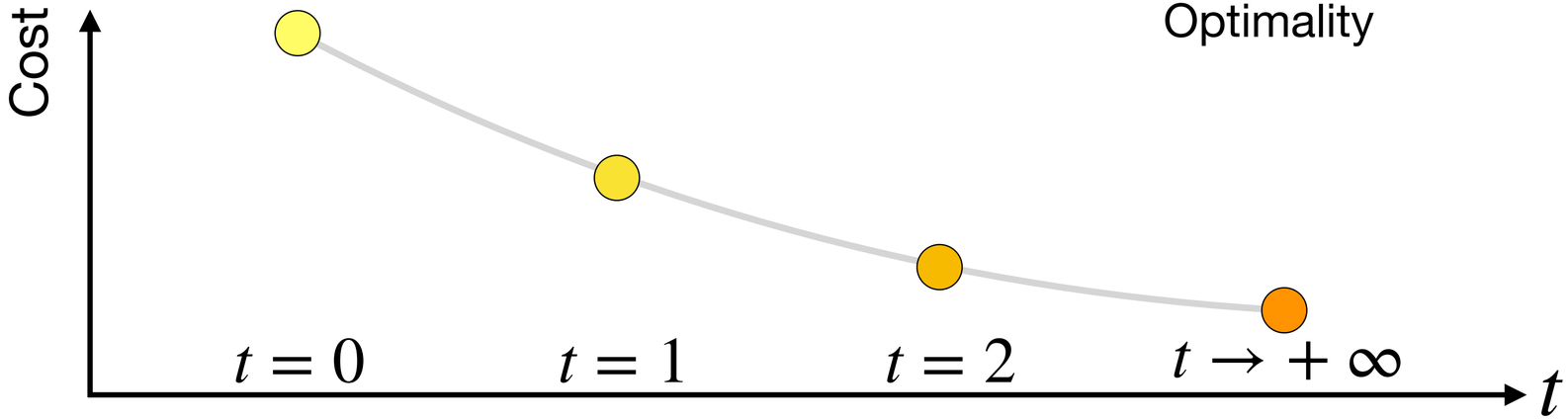


Optimality



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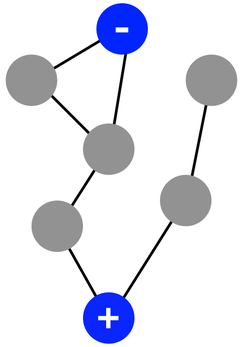

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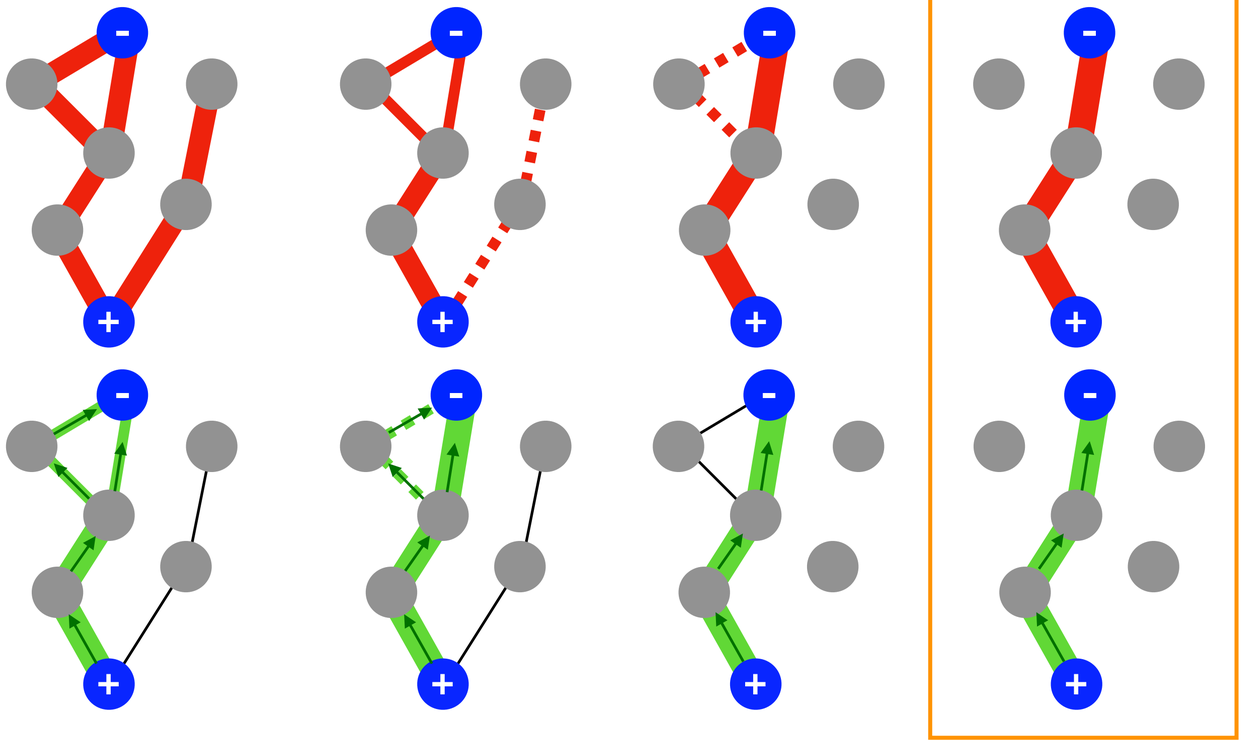
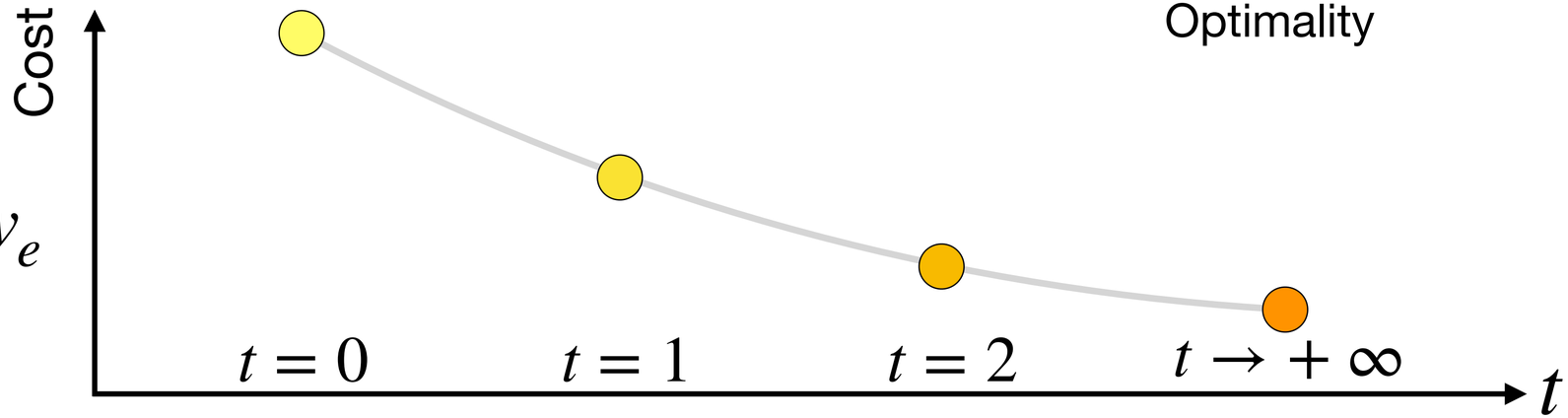
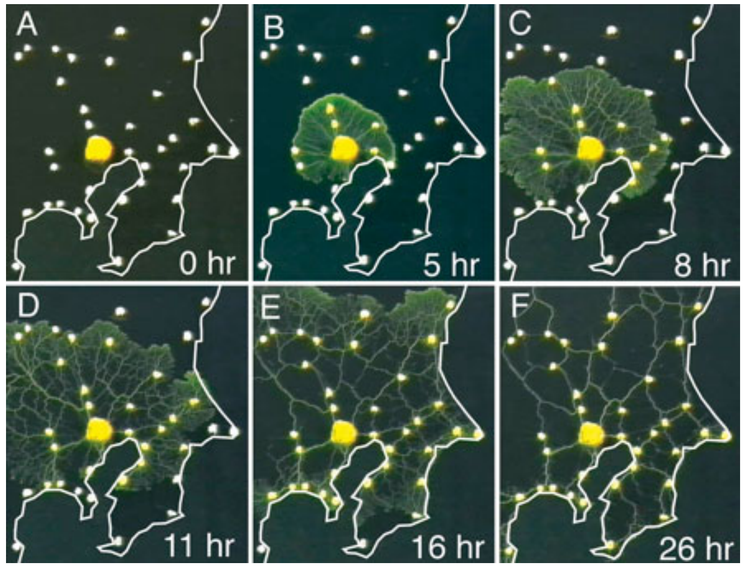


Optimality

$$\begin{cases} \dfrac{d\mu_e}{dt} = f(|F_e|) - \mu_e \\ F_e(\mu, p) = \mu_e (p_u - p_v)/w_e \\ \text{Conservation of mass} \end{cases}$$



Background: summary & minimal references



Tero *et al.*
Science 2010

Physarum can compute shortest paths

☆

Vincenzo Bonifaci [a 1], Kurt Mehlhorn [b], Girish Varma [c 1]

Bonifaci *et al.*
J. Theor. Bio. 2012

Numerical Solution of Monge–Kantorovich Equations via a Dynamic Formulation

Enrico Facca[1] · Sara Daneri[2] · Franco Cardin[3] · Mario Putti[3]

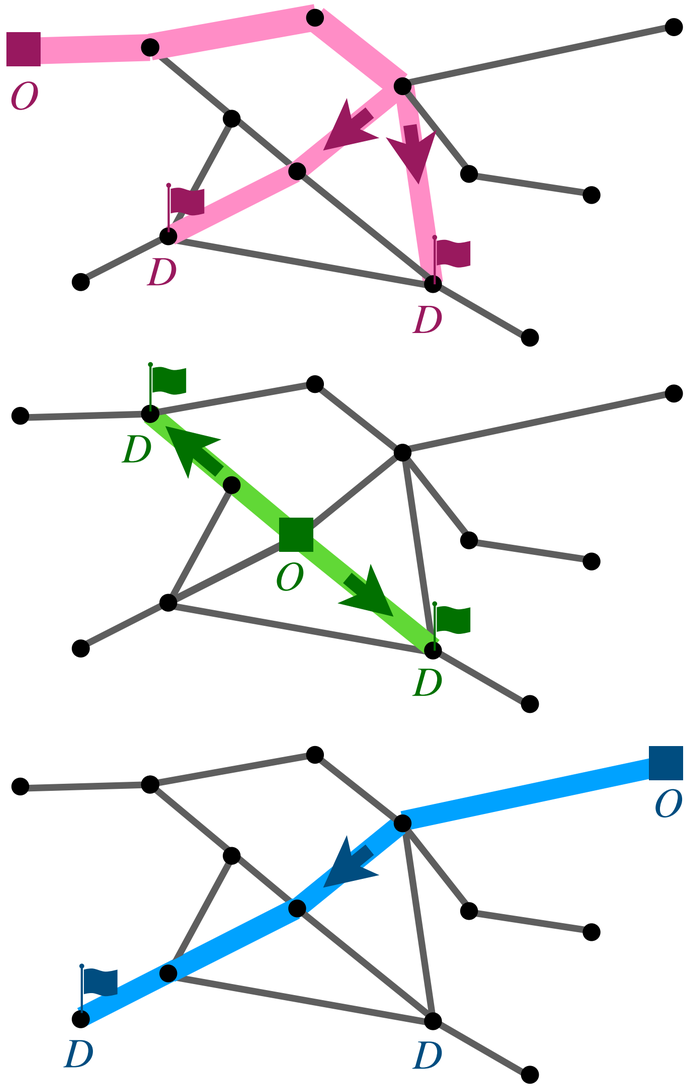
Facca *et al.*
J. Sci. Comp. 2020

Problem & motivation: traffic congestion

nlc.org

Goal: design network infrastructures that enable **efficient transport** (shortest path) while **mitigating traffic congestion** (robustness)

Problem & motivation: traffic congestion



- Passengers travel greedily from one **O**rigin to multiple **D**estinations (Sources and Sinks)

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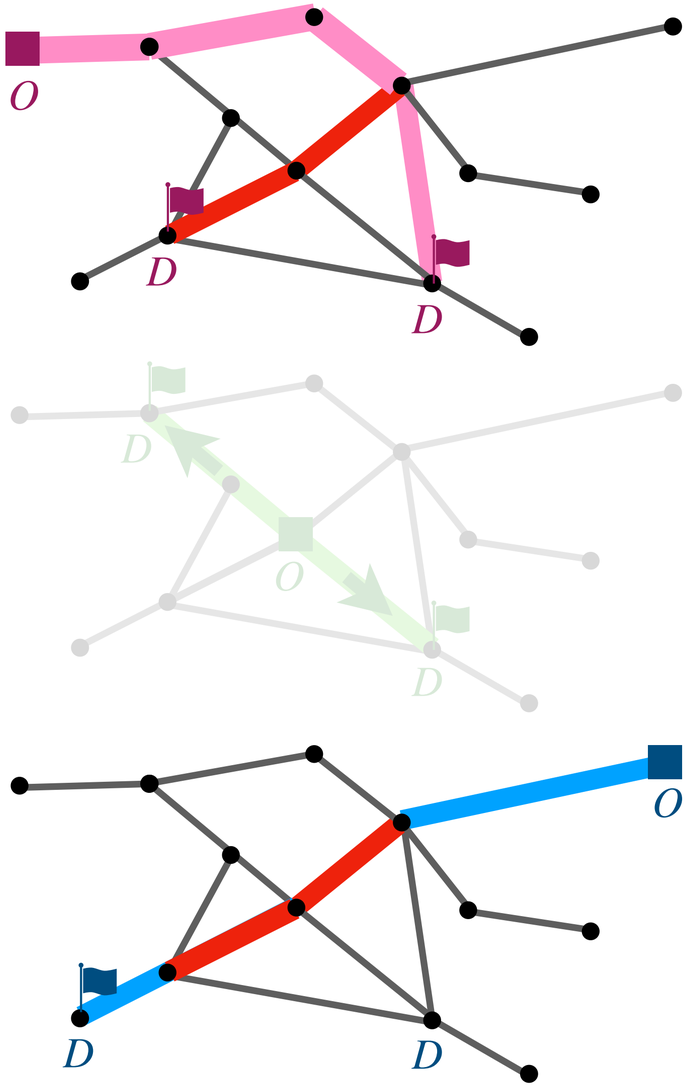


- Passengers travel greedily from one **O**rigin to multiple **D**estinations (Sources and Sinks)

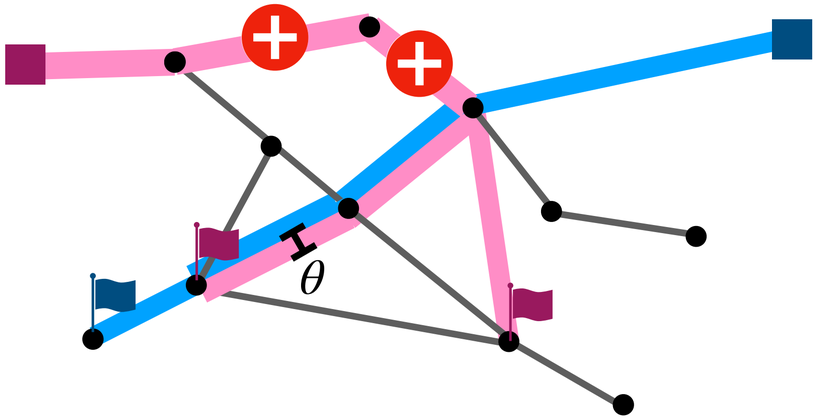
- Passengers trigger **traffic congestion**

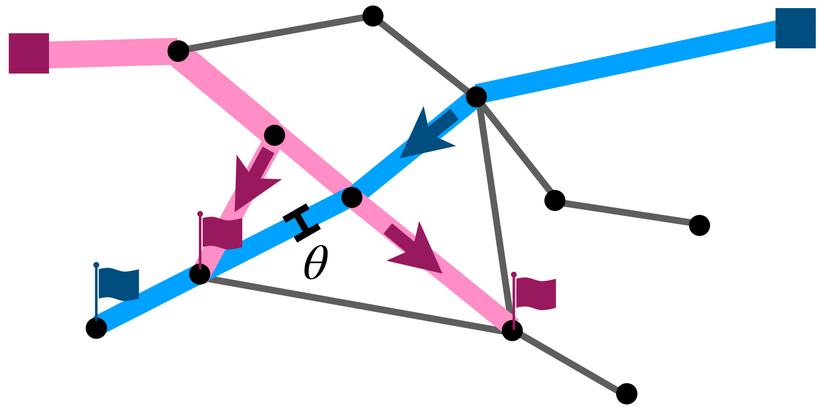
$$\sum_{r} |F_e^r| \geq \theta$$

Bilevel optimization for traffic mitigation: theory

Modeling assumptions:

- A network manager **tunes the edge weights to mitigate traffic**





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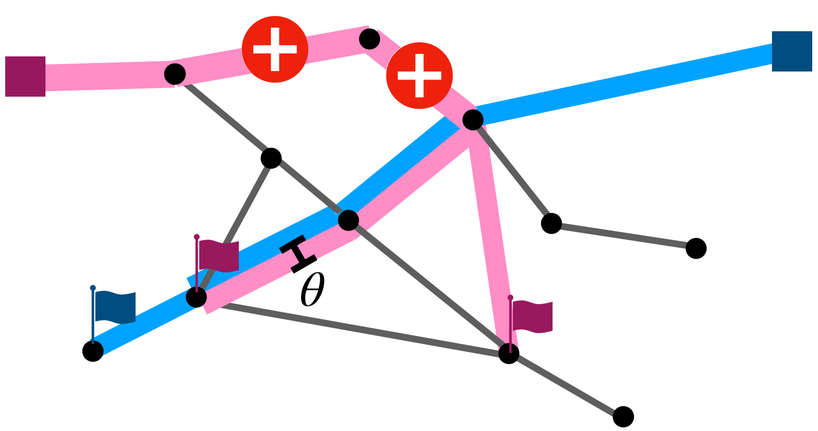

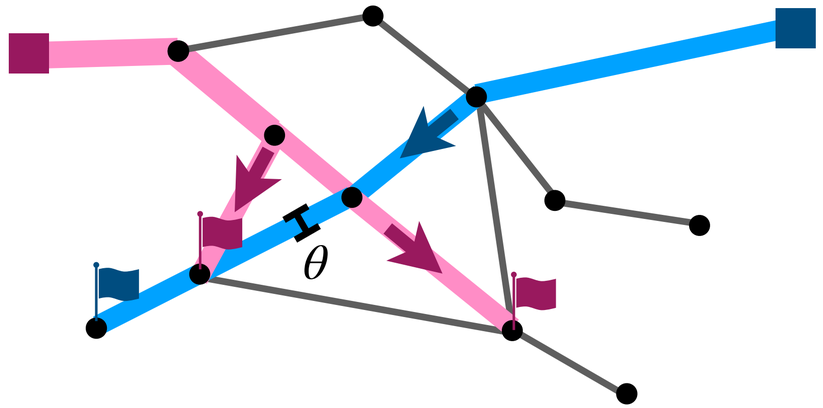

- We pose a **bilevel optimization** problem:

$$\min_{w} \text{CongestionCost}_{\theta}(w; \hat{\mu}) : \hat{\mu} = \text{argmin}_{\mu} \text{TravelCost}(\mu; w)$$

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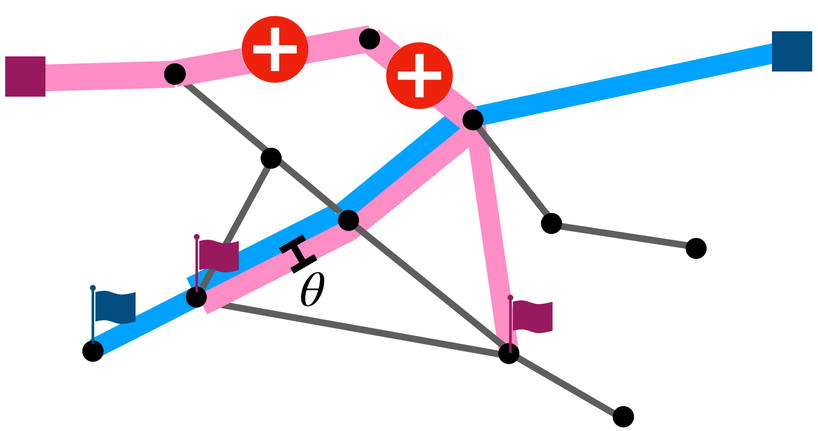

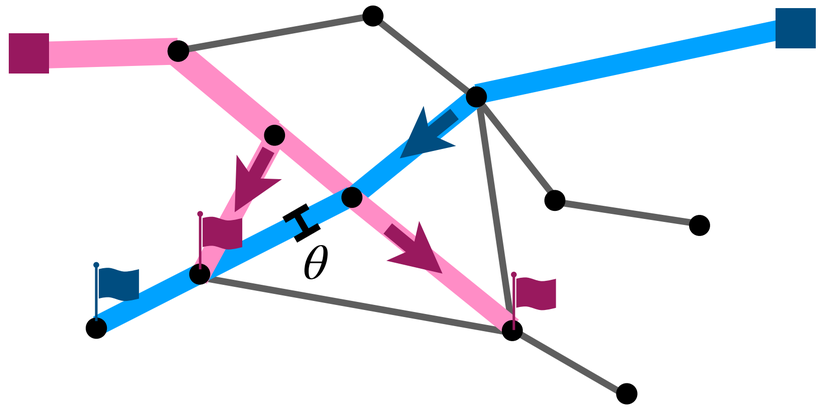

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$$\Delta_e := \sum_r |F_e^r| - \theta \;:\; \Omega_\theta = \sum_e \Delta_e^2 H(\Delta_e) \qquad J = \sum_{e,r} w_e |F_e^r(\mu, w)|$$

Bilevel optimization for traffic mitigation: results

Results:

- **Closed-form adaptation equations** (Lonardi and De Bacco Phys. Rev. Lett. 2023)

$$
\begin{cases}
\dfrac{d\mu_e^r}{dt} = |F_e^r| - \mu_e^r \\
w \leftarrow \text{PGSD}(\text{CongestionCost}_\theta(w; \mu)) \\
F_e^r(\mu, p) = \mu_e (p_u - p_v)/w_e \\
\text{Conservation of mass}
\end{cases}
$$

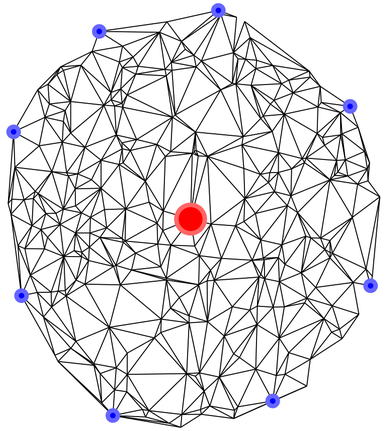
Bilevel optimization for traffic mitigation: results

Results:

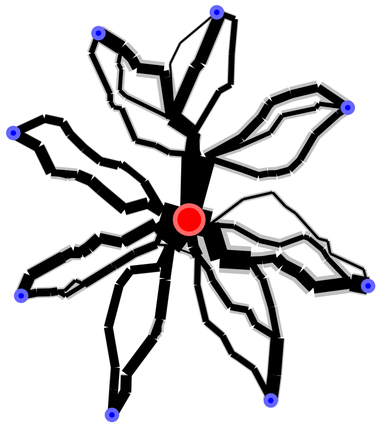
- **Closed-form adaptation equations** (Lonardi and De Bacco Phys. Rev. Lett. 2023)

$$\begin{cases} \dfrac{d\mu_e^r}{dt} = |F_e^r| - \mu_e^r \\ w \leftarrow \text{PGSD}(\text{CongestionCost}_\theta(w; \mu)) \end{cases}$$

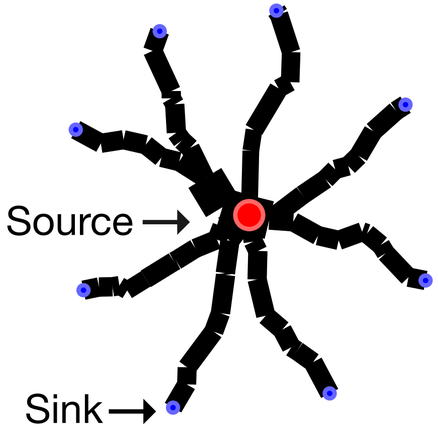
- Systematic exploration of **congestion regimes** (Lonardi and De Bacco Phys. Rev. Lett. 2023)



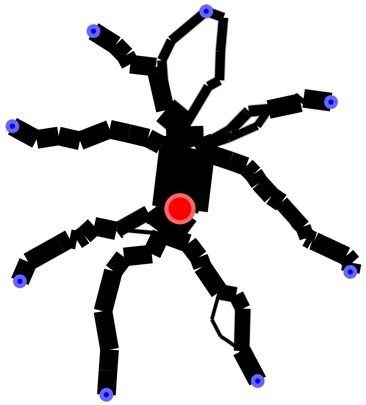
Topology



Bilevel optimization



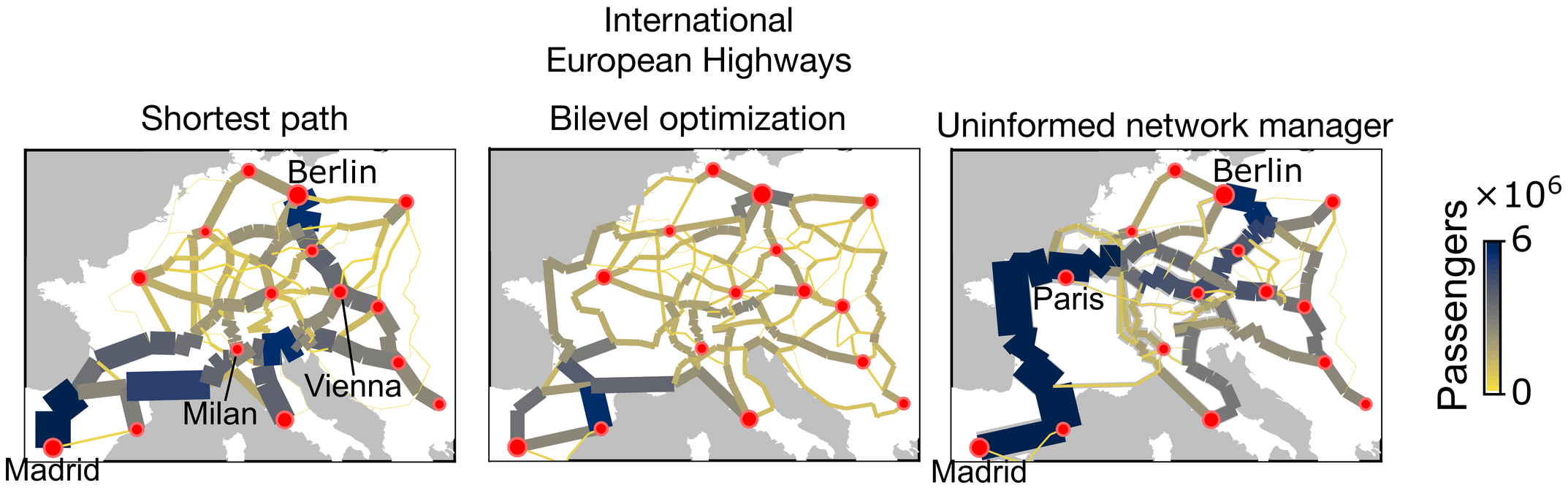
Shortest path



Uninformed network manager

Bilevel optimization for traffic mitigation: urban transportation

- Bilevel optimization scheme returns **shorter travel times** on real-world networks (Lonardi and De Bacco Phys. Rev. Lett. 2023)



Lonardi and De Bacco Phys. Rev. Lett. 2023

BROT

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Phys. Rev. Lett. **131**, 267401 – Published 26 December 2023

Thank You!
Q&A

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