Lonardi and De Bacco arXiv:2306.16246 (2023)

Bilevel Optimization for Traffic Mitigation in Optimal Transport Networks

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Max Planck Institute for Intelligent Systems

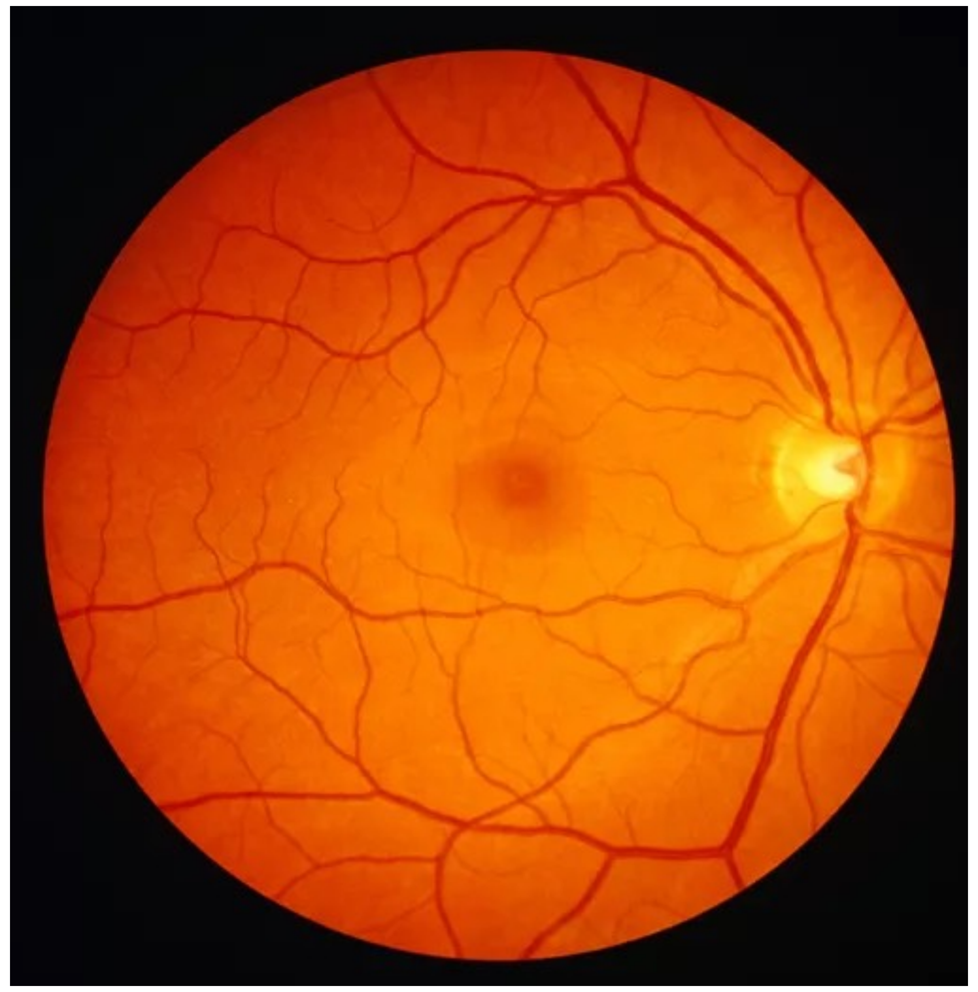
MAX-PLANCK-GESELLSCHAFT

impрs-is

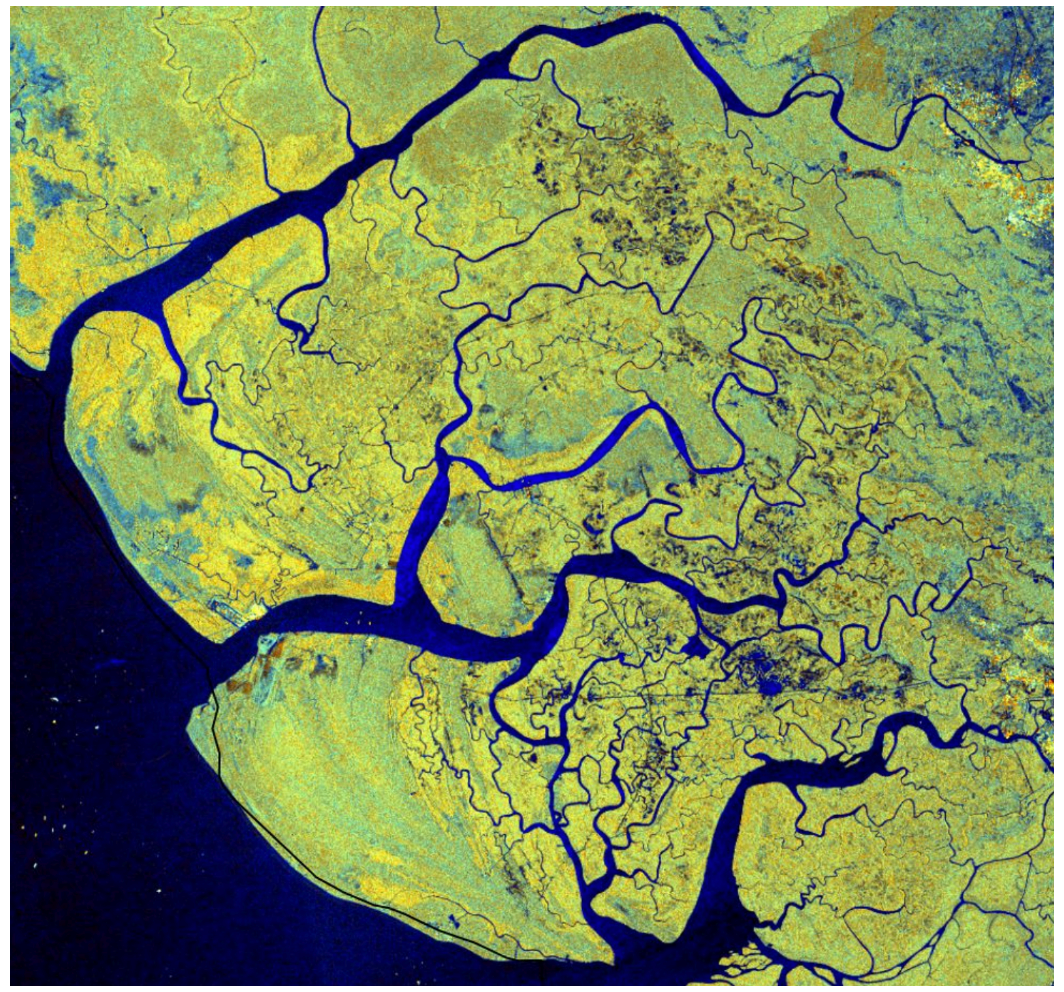
Cyber Valley

Motivation

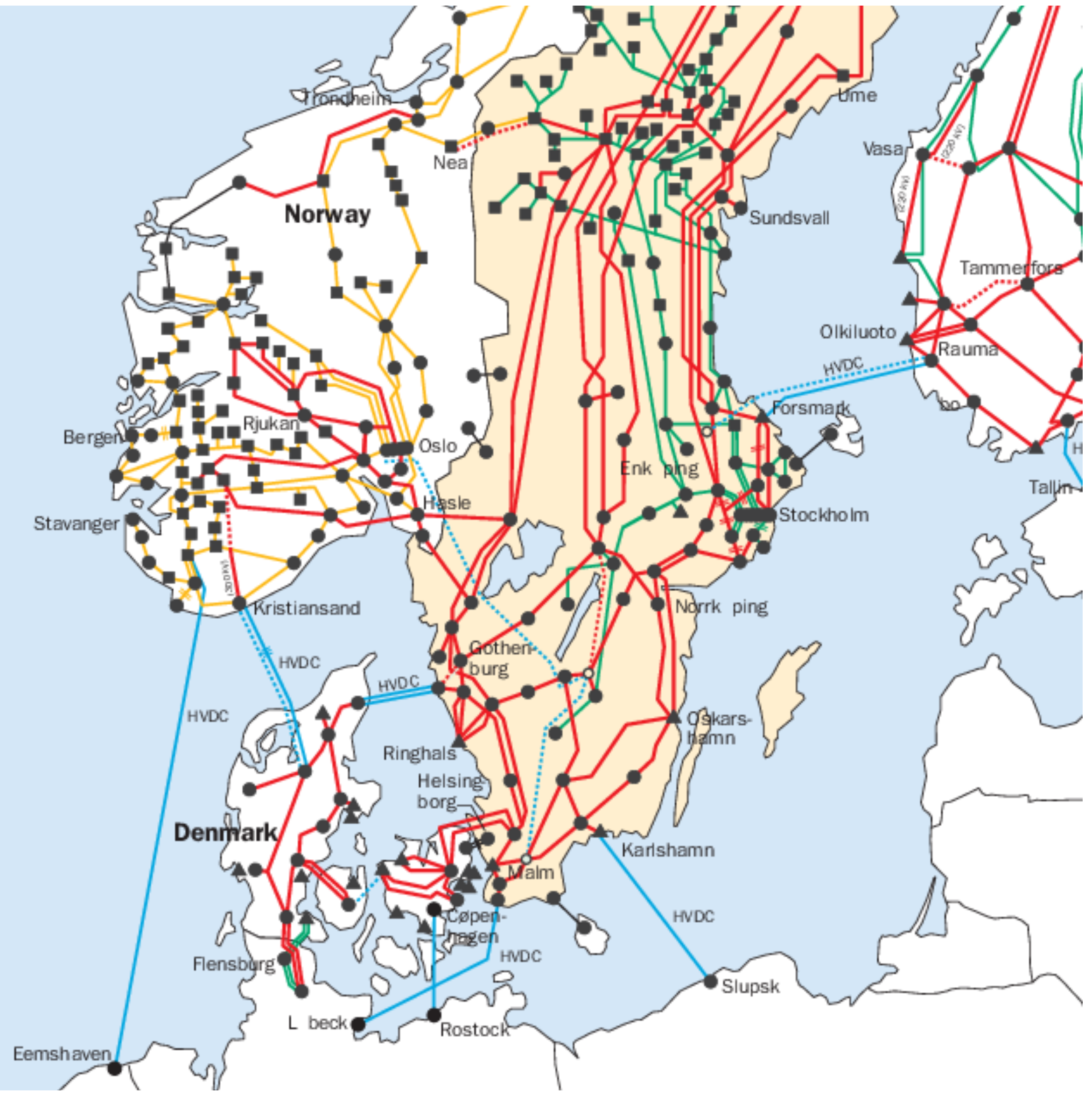
Transport networks are pervasive at different scales



UHB Trust

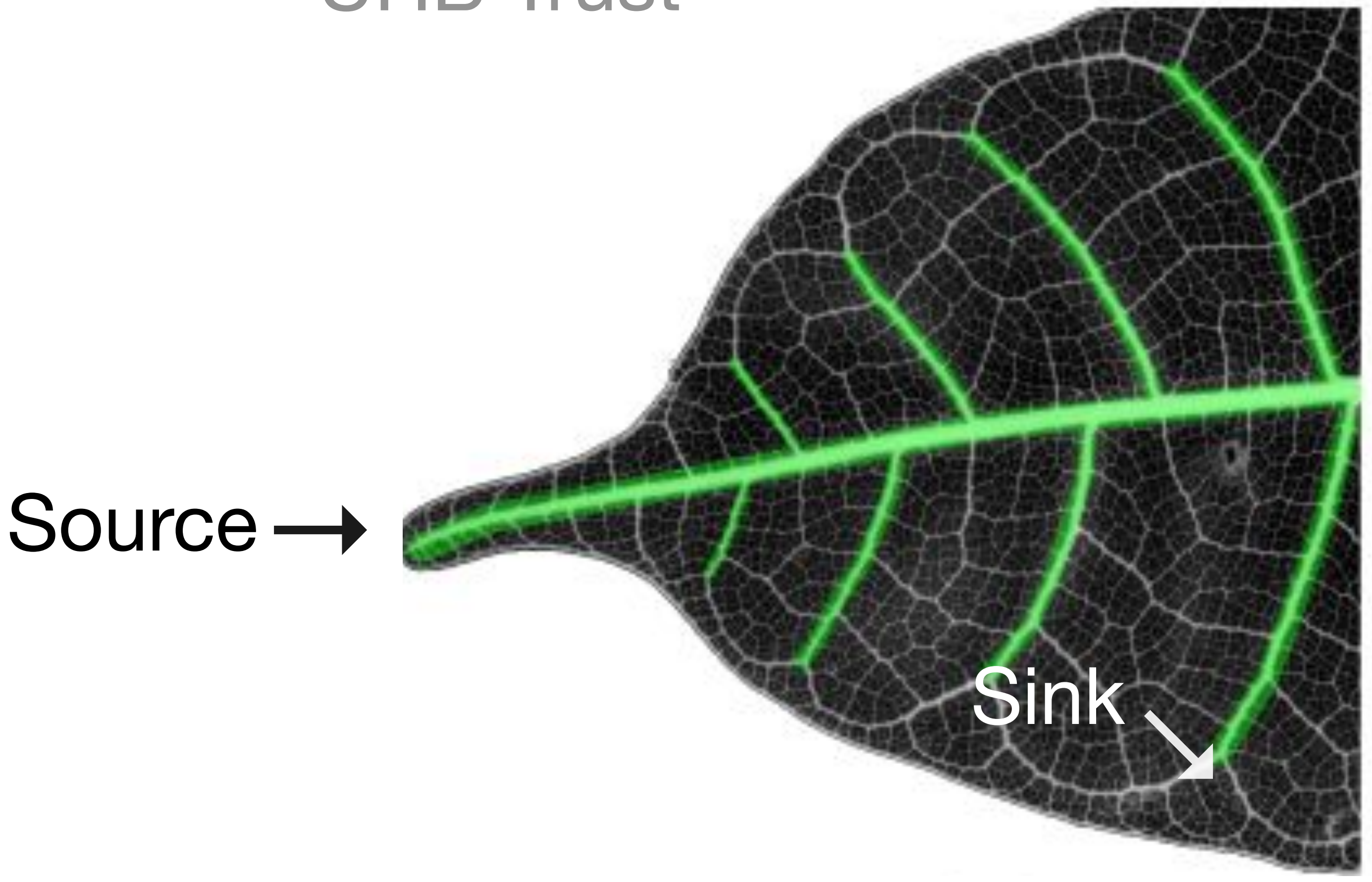


ESA



Perninge
KTH (2011)

Transport for
London

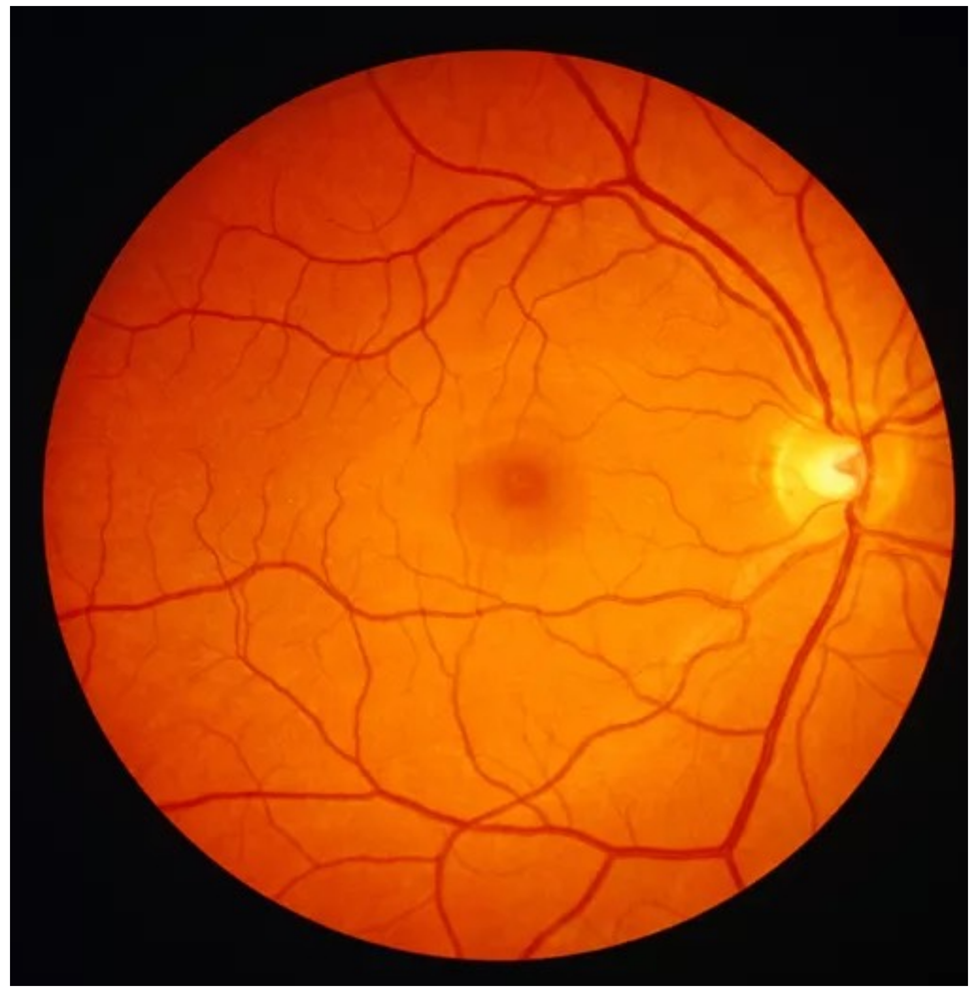


Ronellenfitsch and Katifori
PRL (2016)

Motivation

Transport networks are pervasive at different scales

Natural systems



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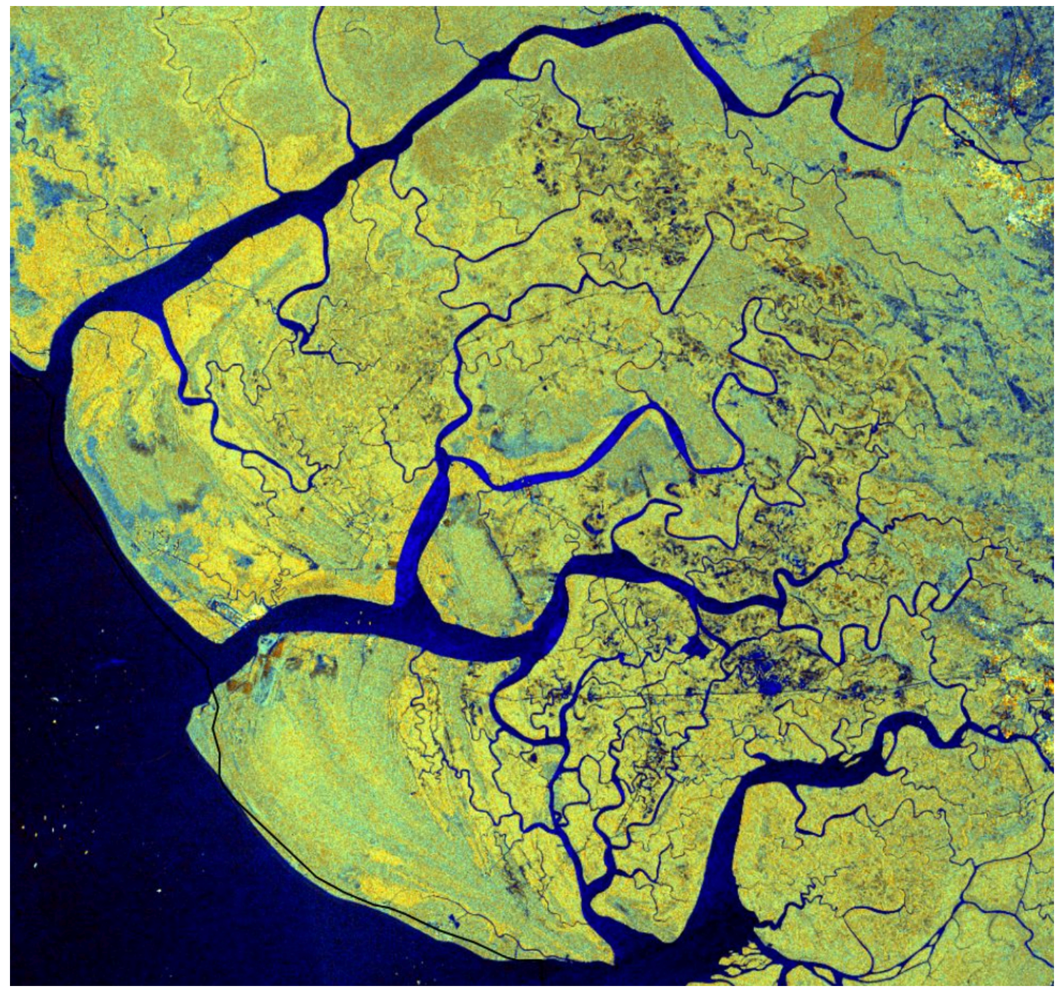

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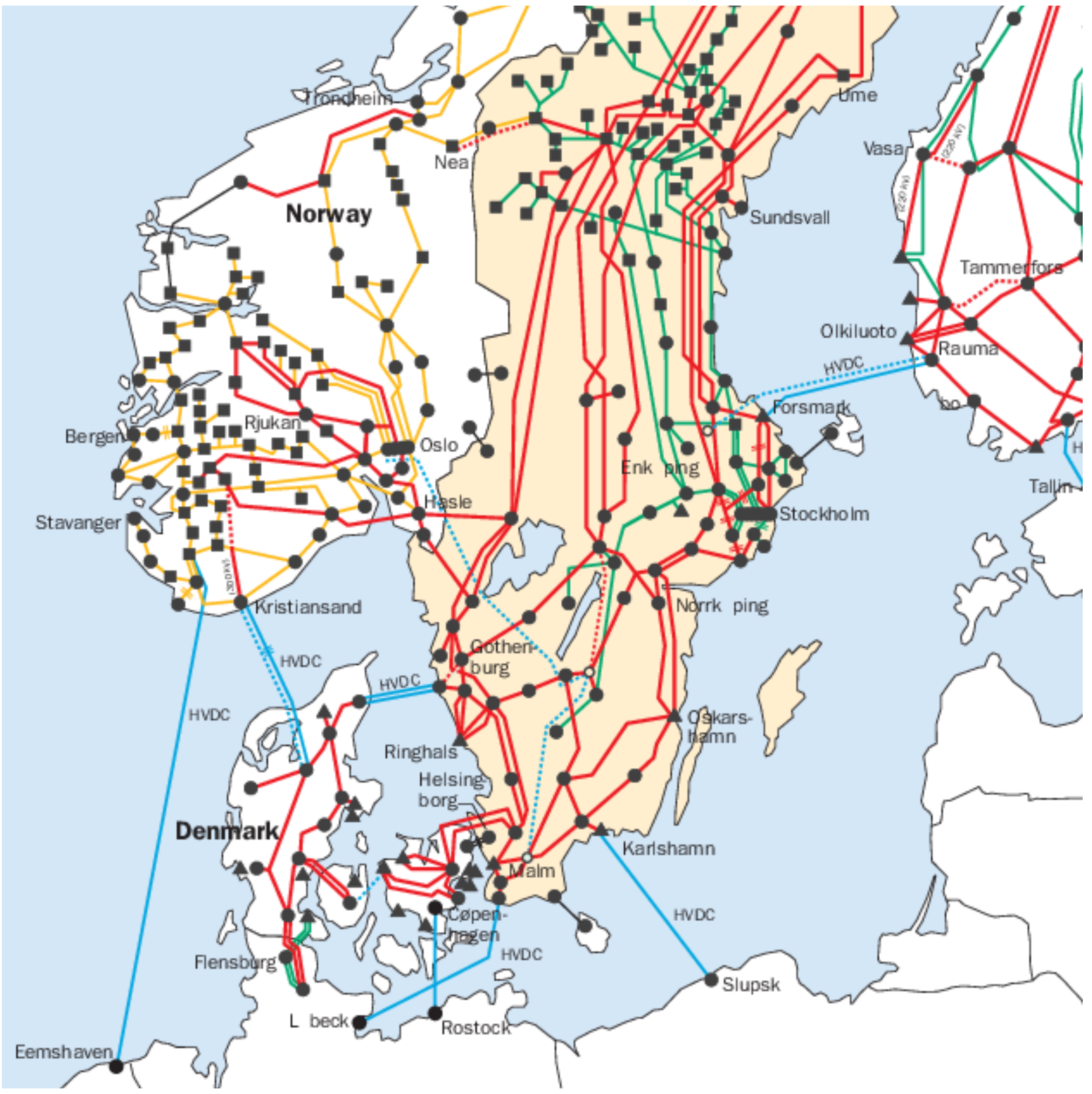

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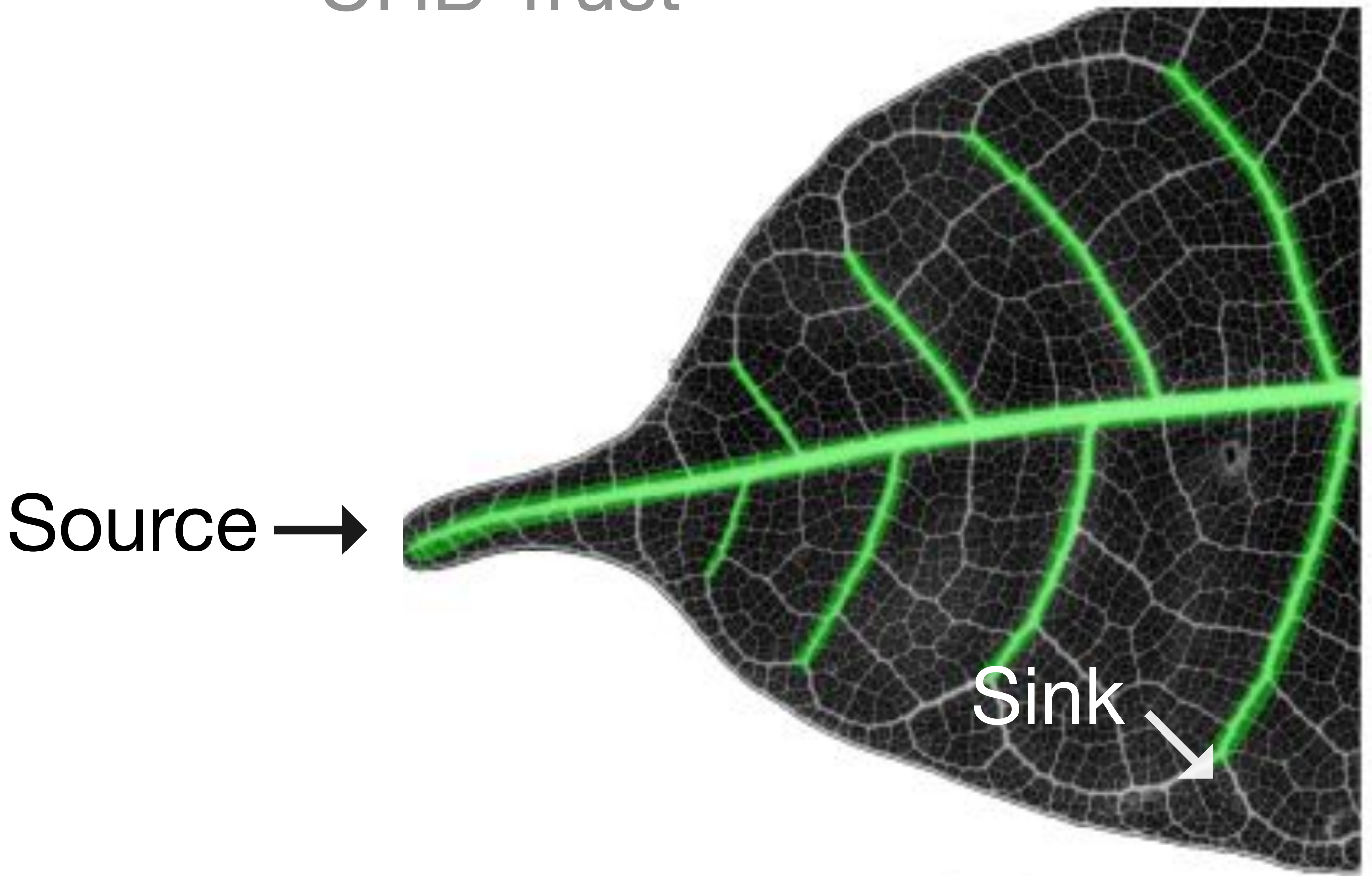

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Adaptation leads to the emergence of
macroscopic properties

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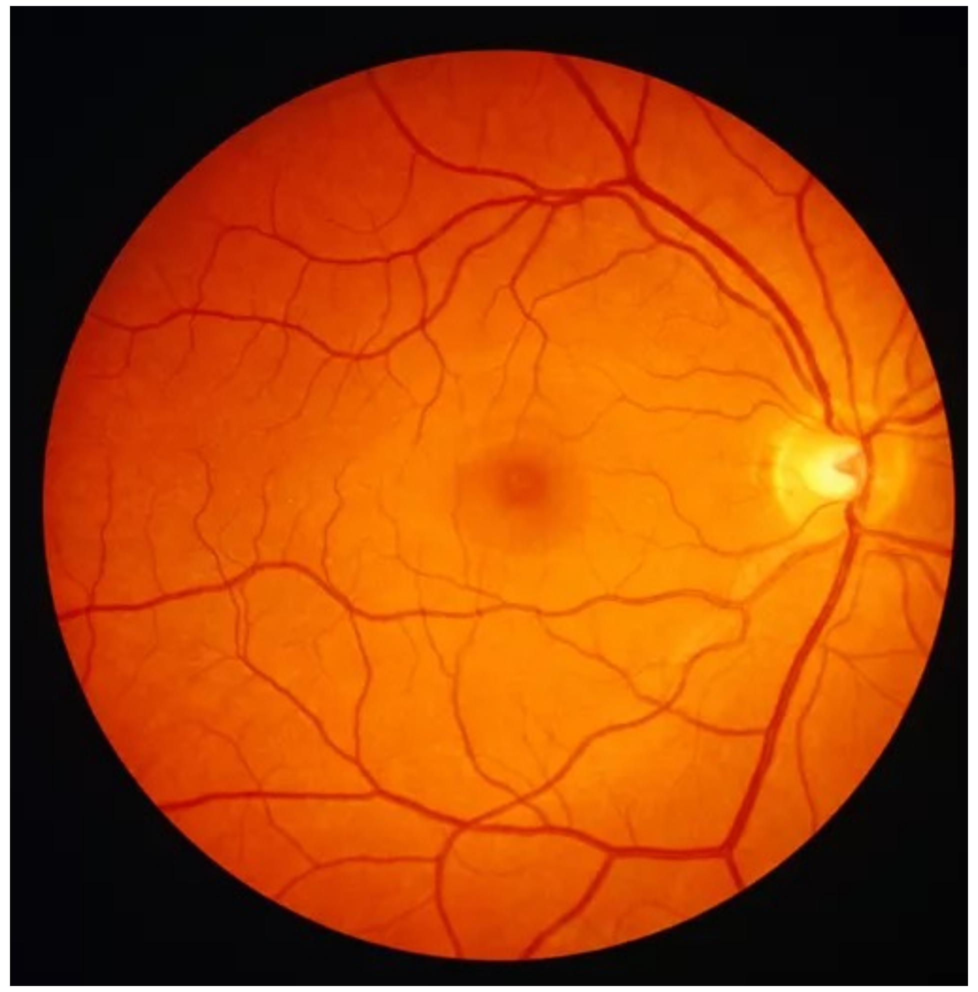

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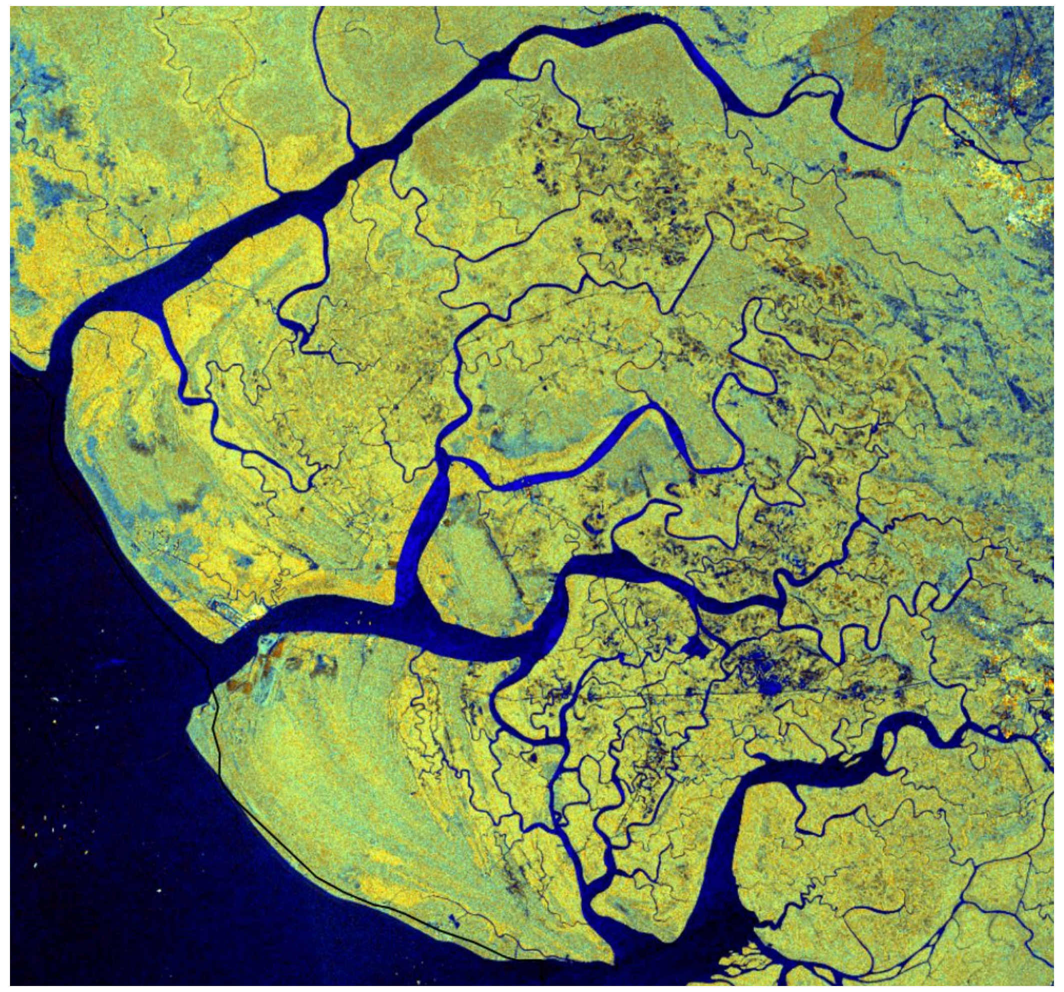
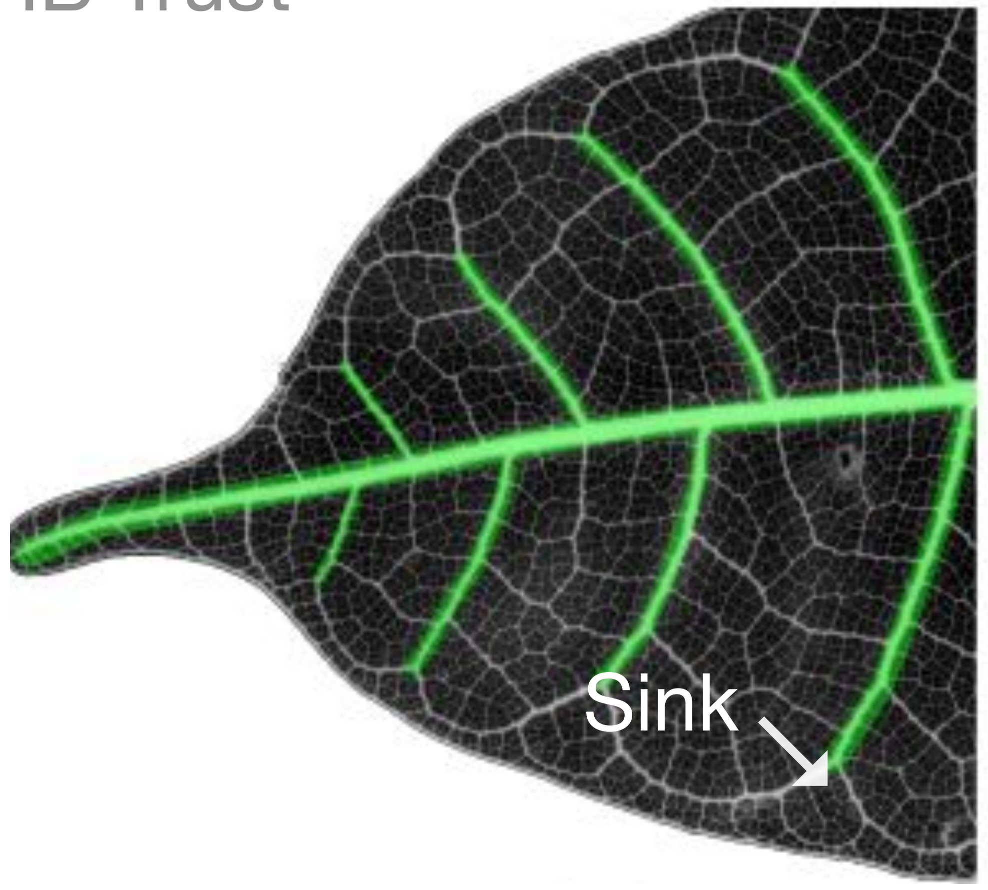


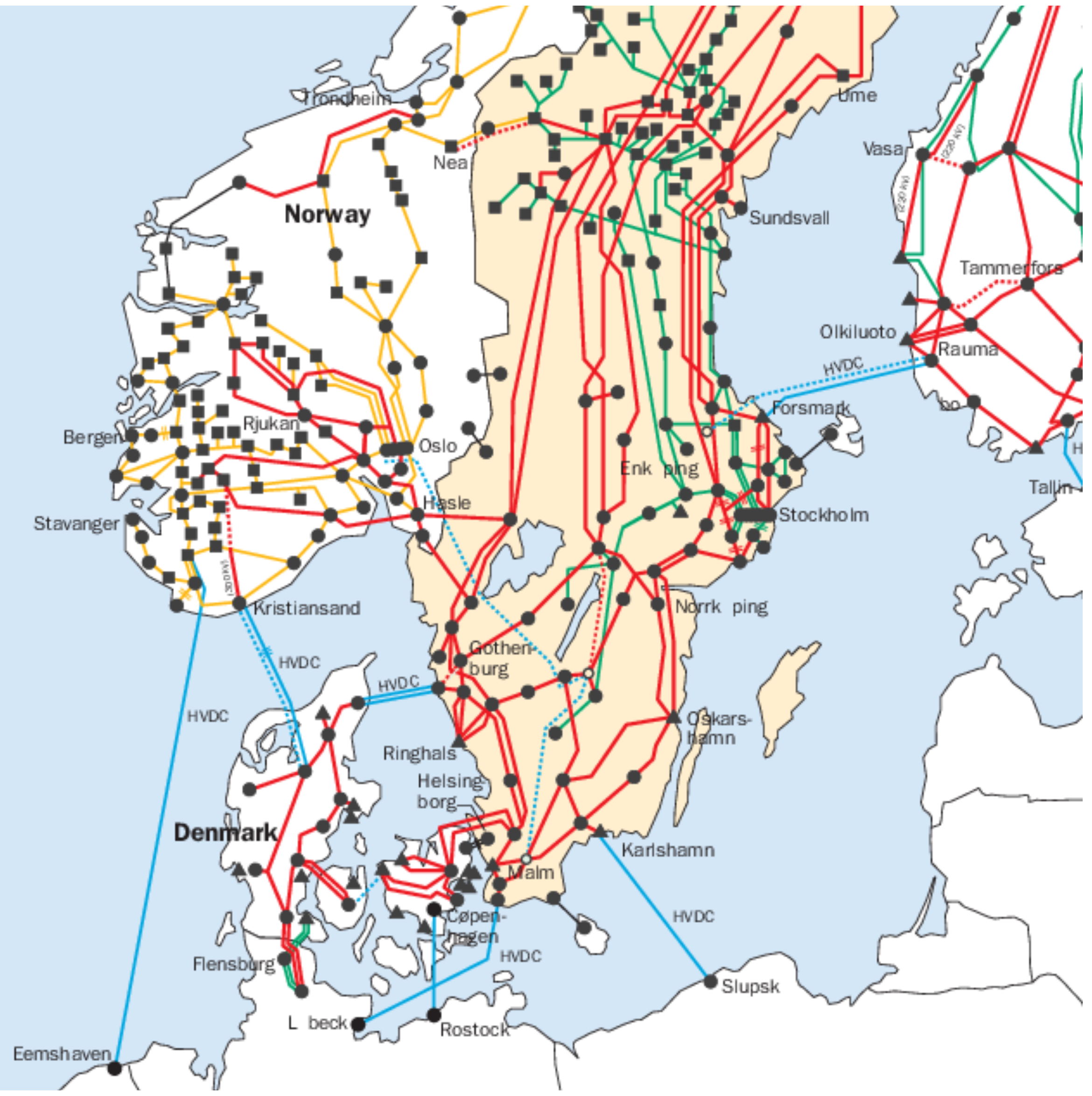
ESA

Source →



Ronellenfitsch and Katifori
PRL (2016)

Artificial systems



Perninge
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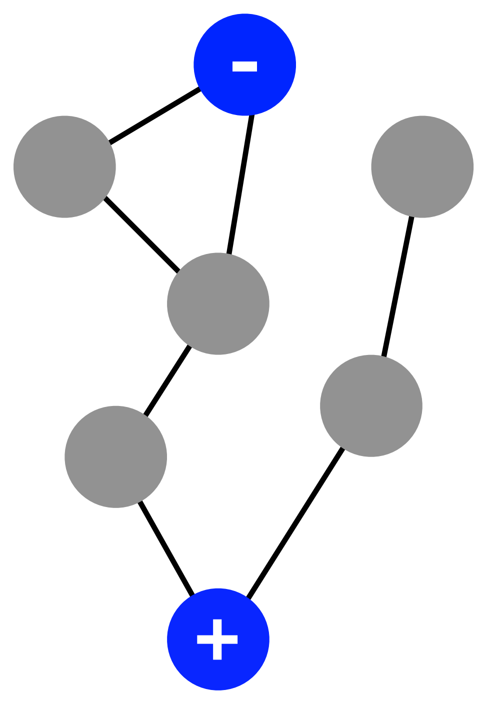
Transport for
London

Adaptation leads to the emergence of macroscopic properties

Idea: leverage adaptation to design urban transportation

Background



μ_e : road capacity

F_e : load displacement

$$\begin{cases} \dfrac{d\mu_e}{dt} = \dfrac{f(|F_e|)}{w_e} - \mu_e \\ \text{Kirchhoff's law} \end{cases}$$

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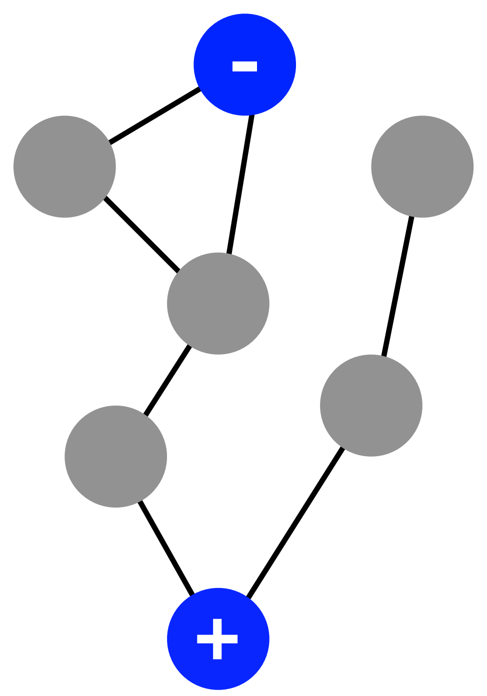

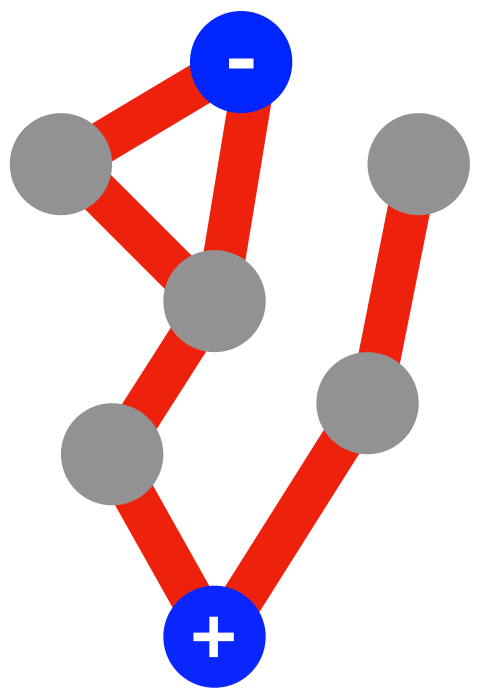

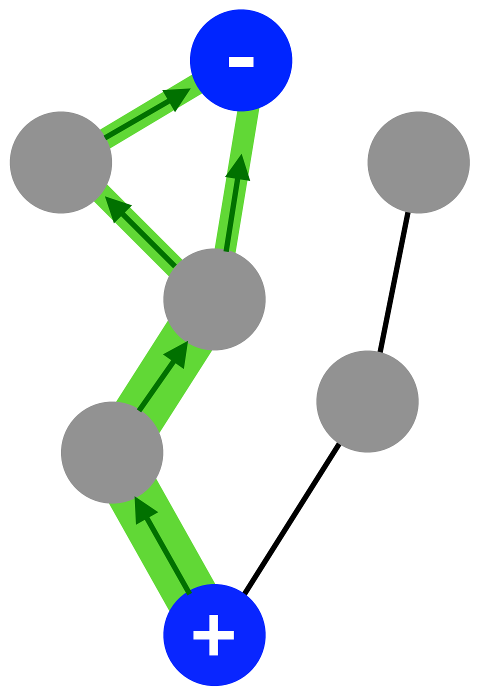

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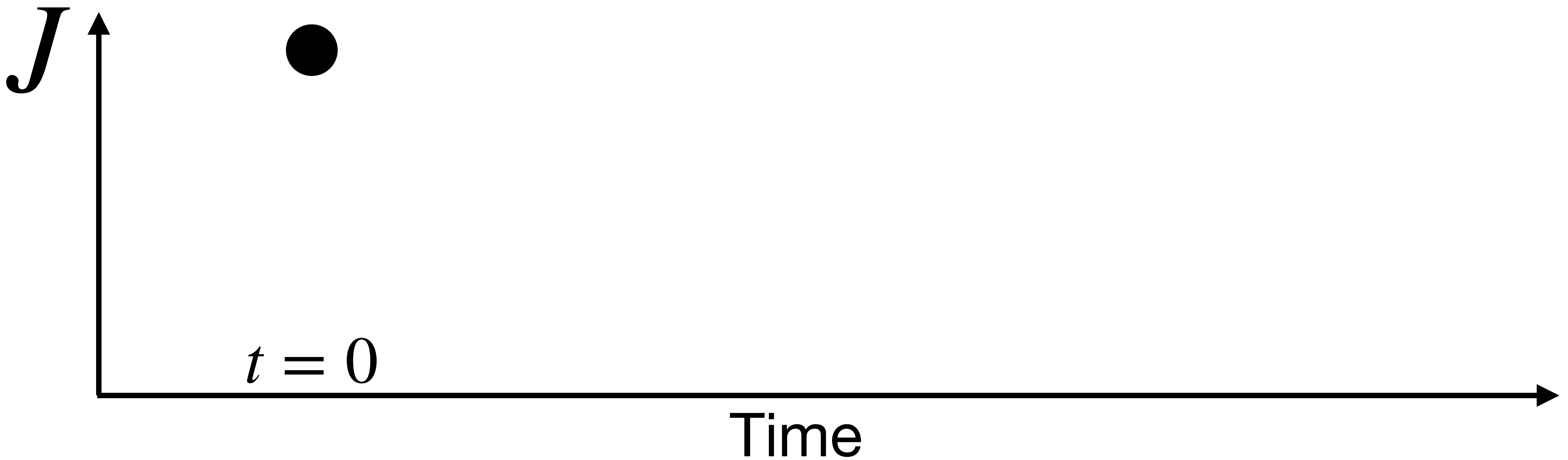

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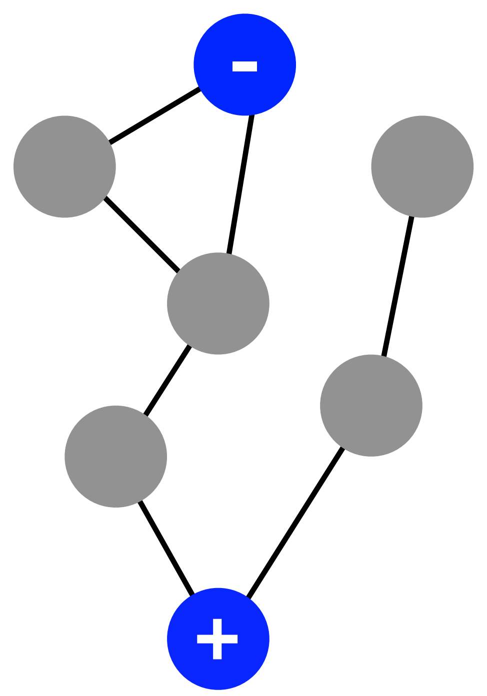

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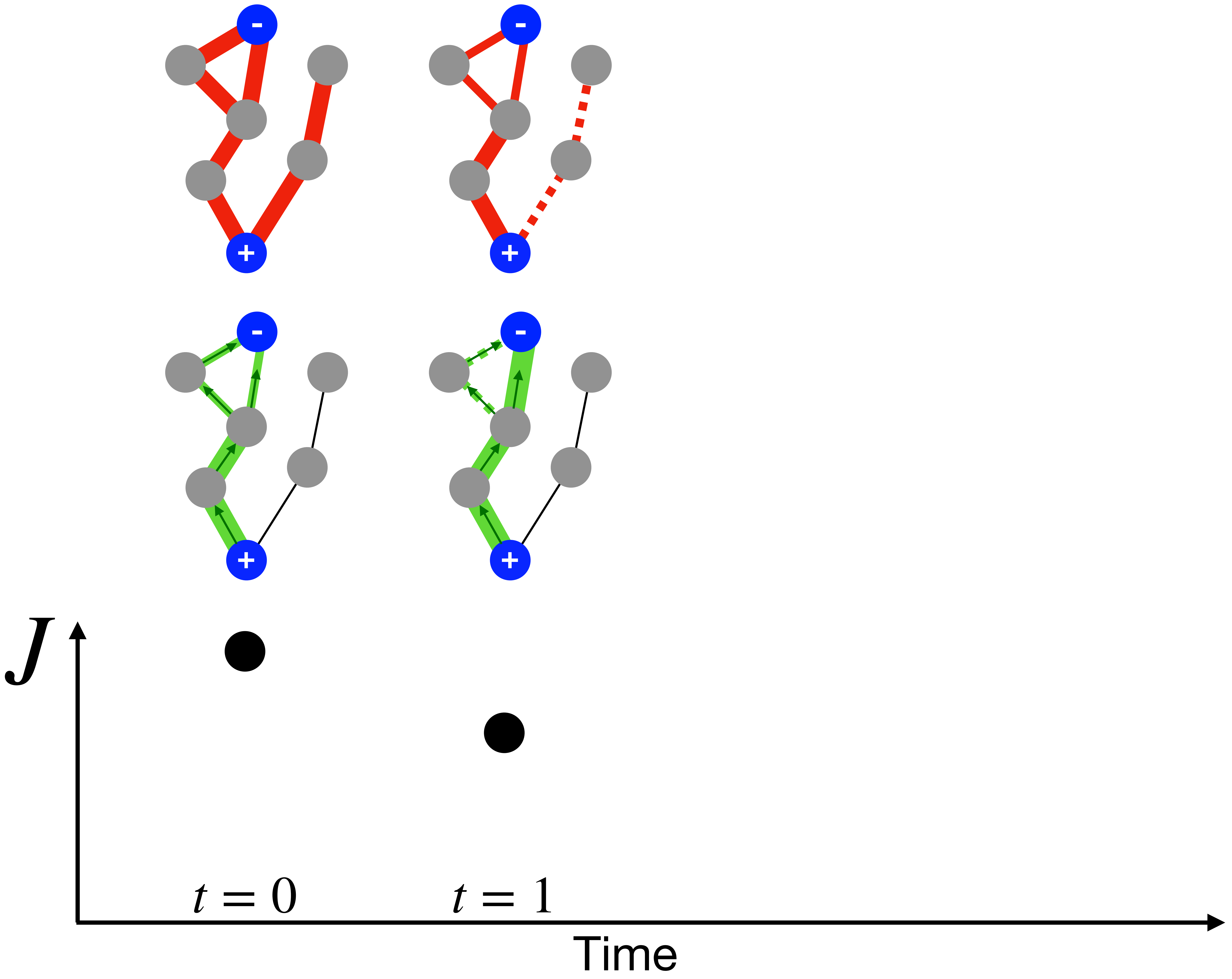

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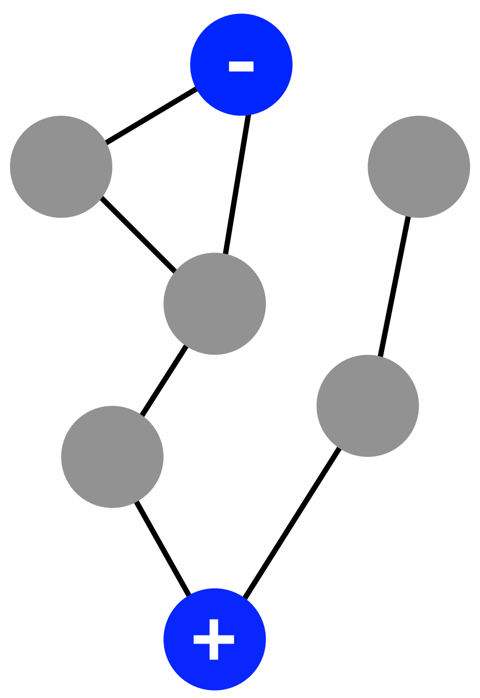

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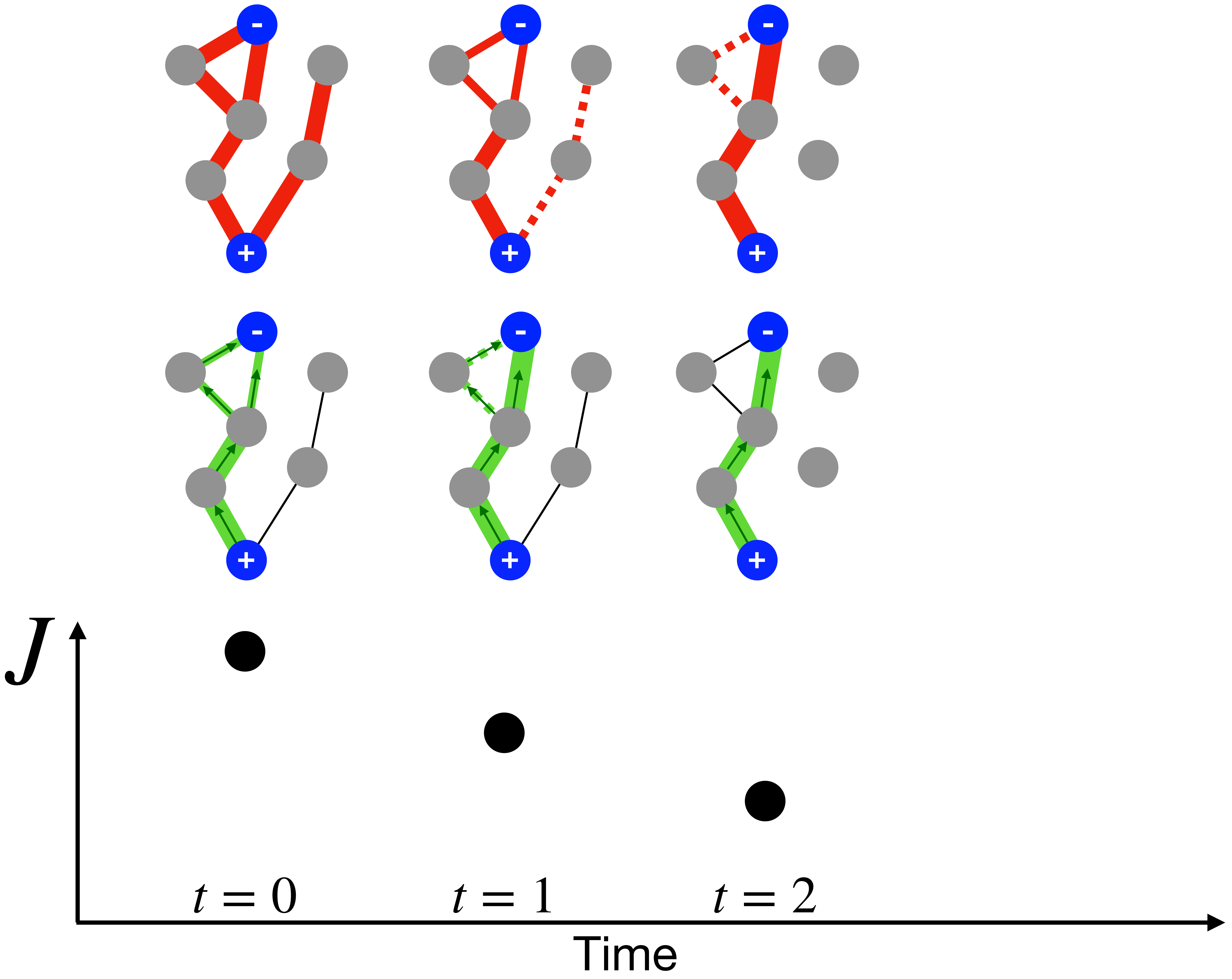

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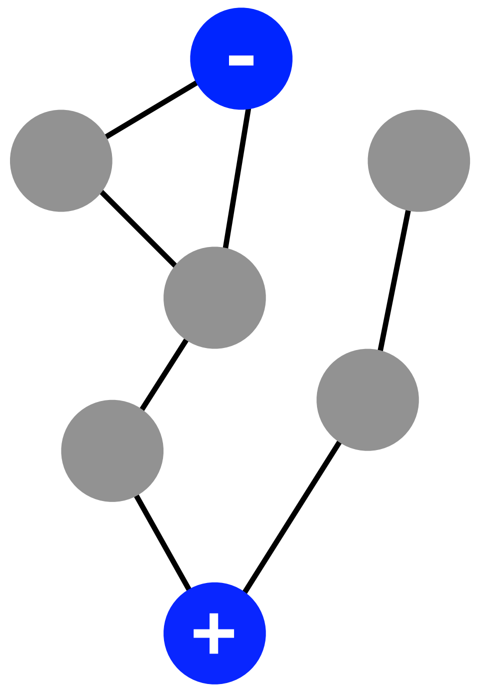

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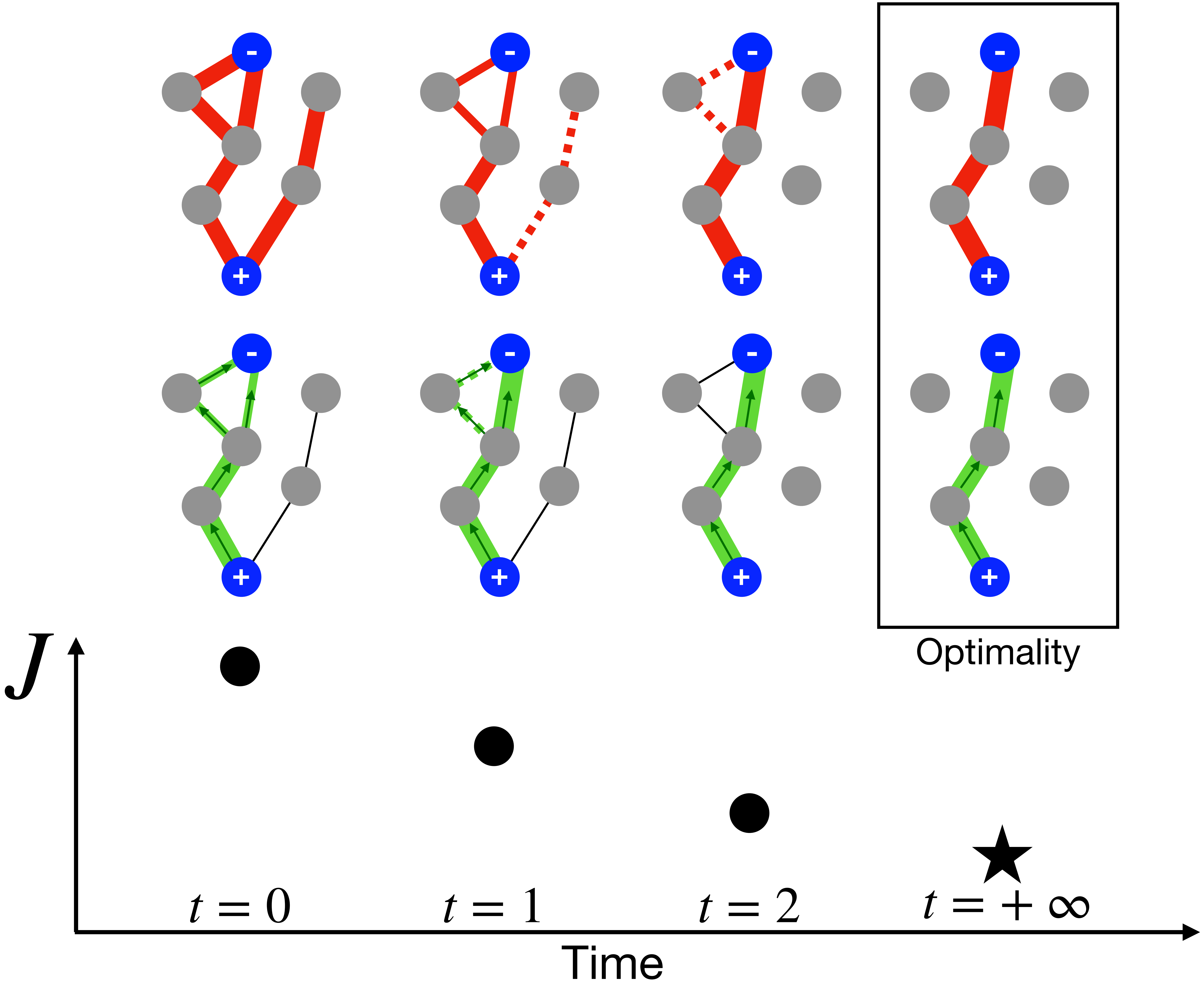
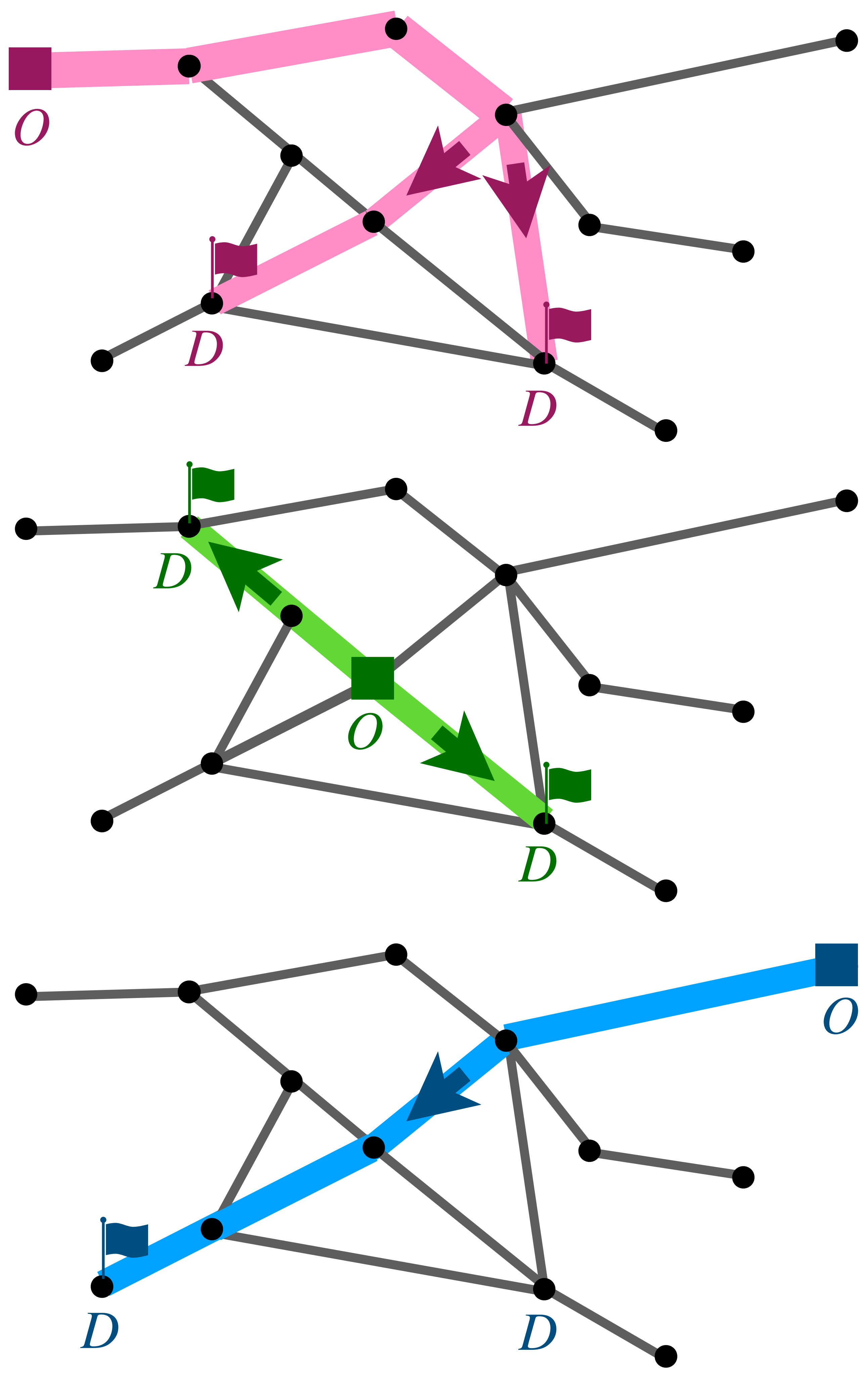


Modeling assumption

Passengers travel from multiple Origins to multiple Destinations



1) Each group of passengers moves greedily from its O to its D
(Wardrop's first principle)

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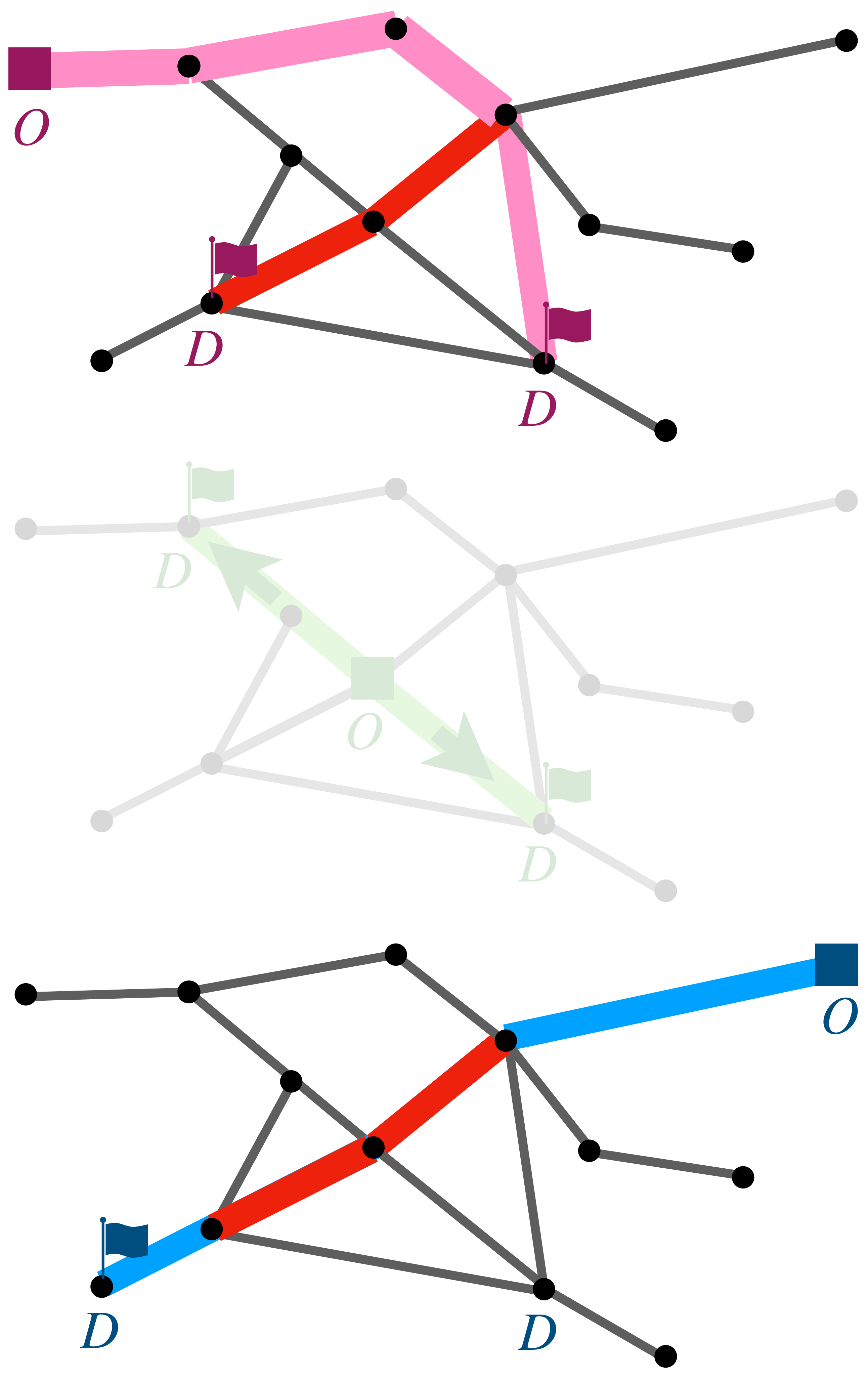


1) Each group of passengers moves greedily from its O to its D (Wardrop's first principle)

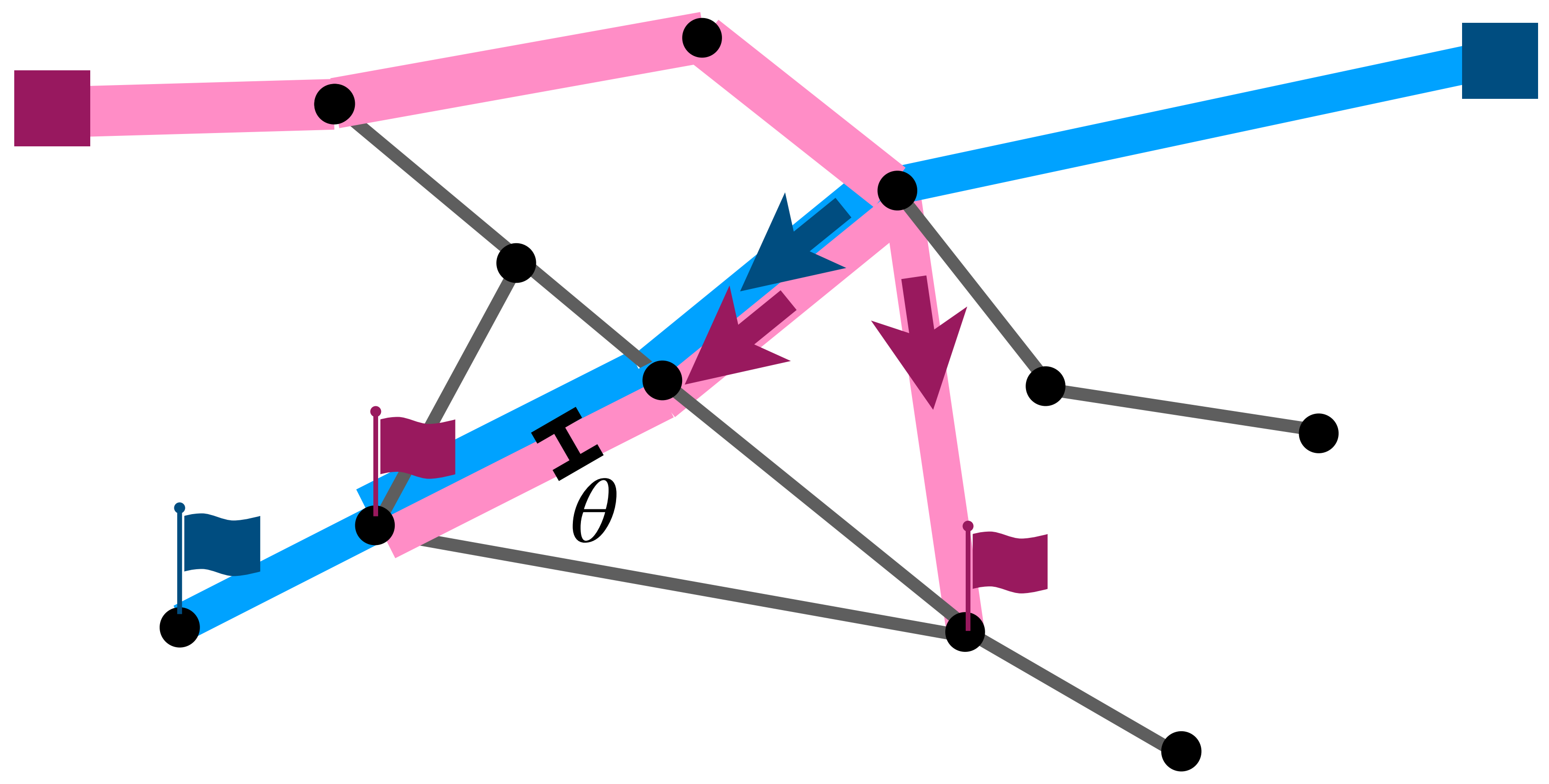
2) Passengers' interaction triggers traffic congestion

Often neglected by adaptation models!

Modeling assumption

Network managers tunes edge weights to mitigate traffic

$$J_e = w_e F_e \quad \Omega_e = F_e^2 H(F_e - \theta)$$



$[w_e] = \$$

$[\theta] =$ # number of people

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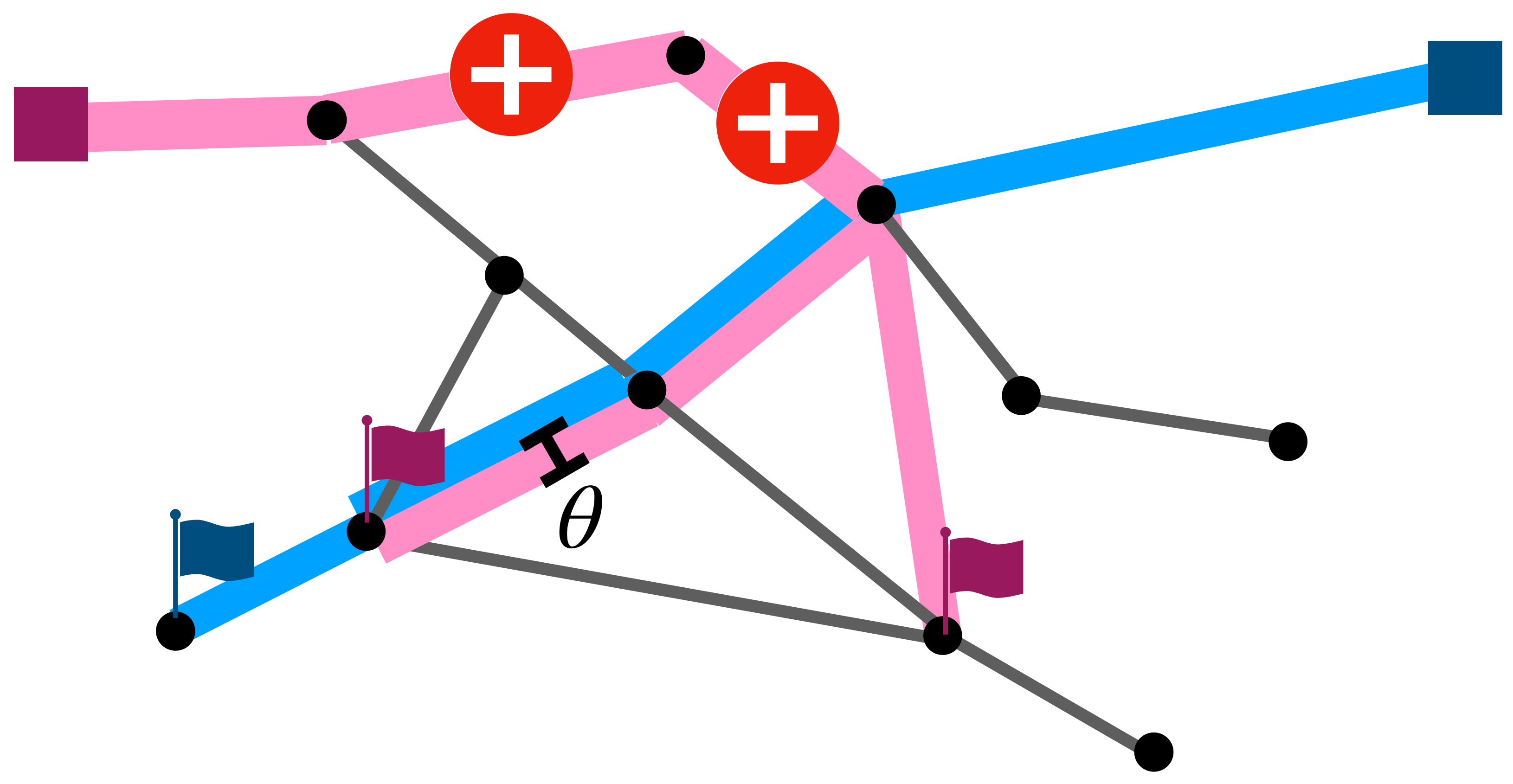

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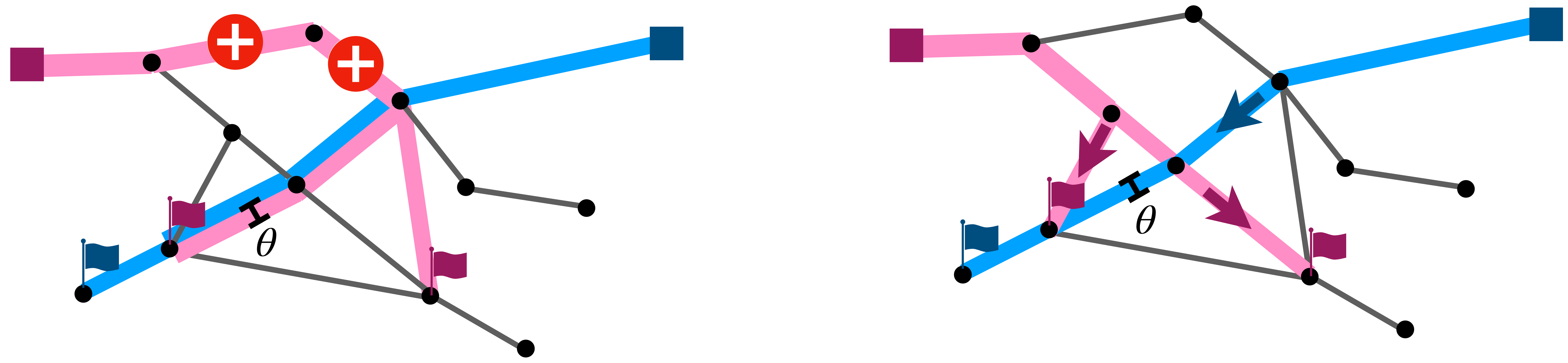

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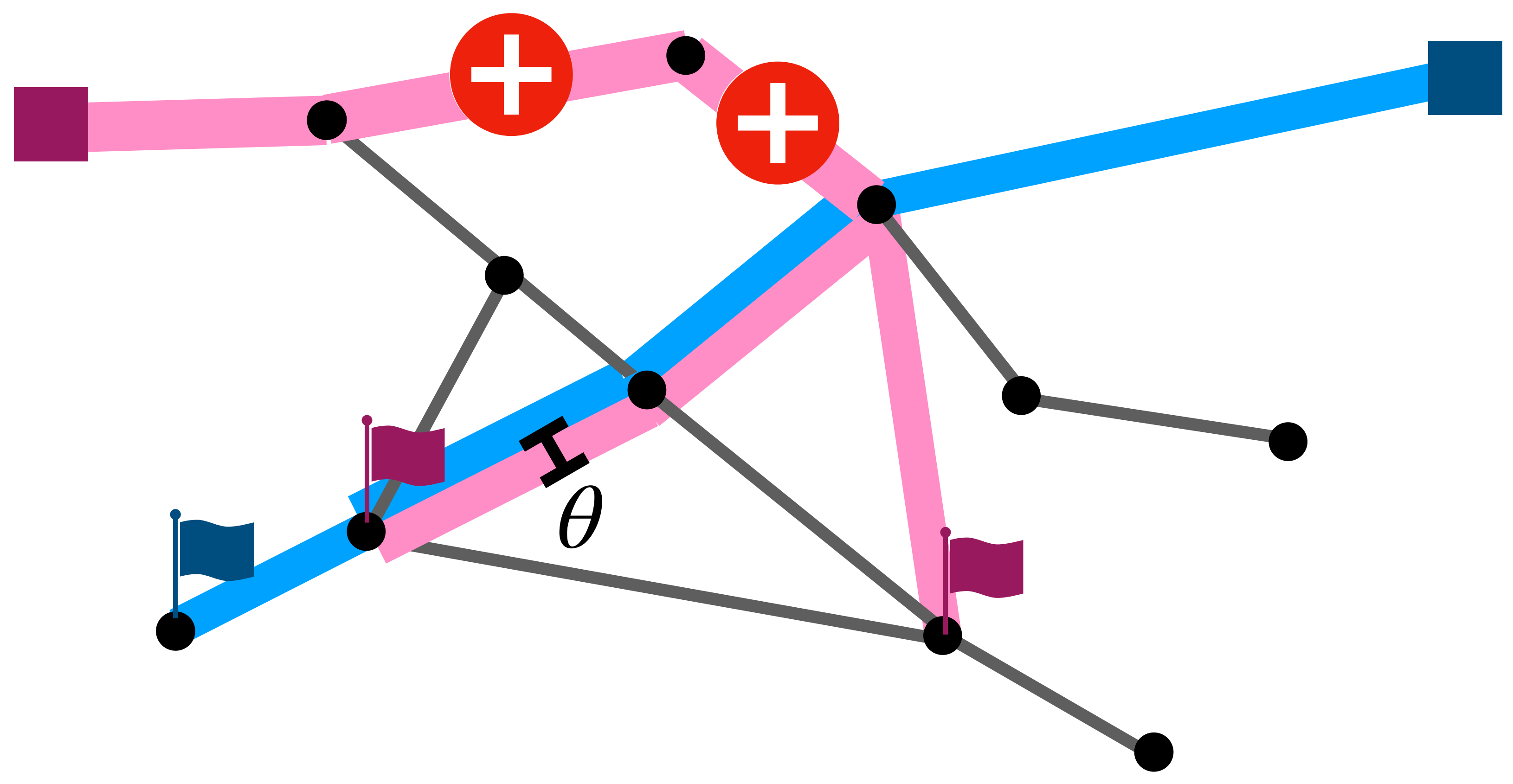

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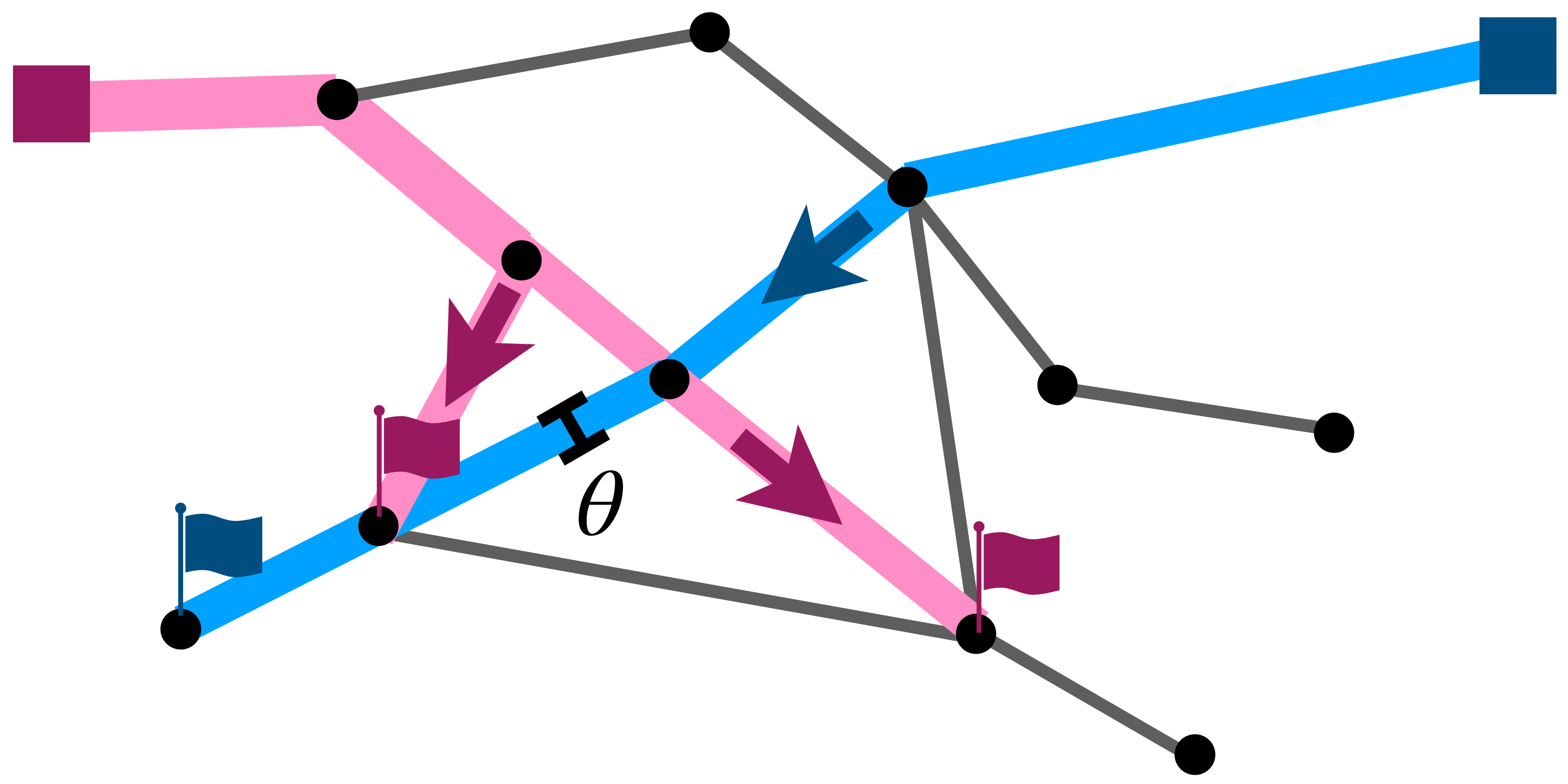

Trade off traffic congestion against transport cost

Research questions

Framing as a bilevel optimization problem

$$\min_{w} \Omega(w; \hat{\mu})$$

$$\hat{\mu} = \operatorname{argmin}_{\mu} J(\mu; w)$$

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Framing as a bilevel optimization problem

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1) Can we find adaptation rules to solve the bilevel optimization problem?

2) Does adaptation shed light on transport network properties?

Results

Contribution 1: Closed-form adaptation rules

$$\min_{w} \Omega(w; \hat{\mu})$$

$$\hat{\mu} = \text{argmin}_{\mu} J(\mu; w)$$

$$\frac{d\mu_e}{dt} = \frac{f(|F_e|)}{w_e} - \mu_e$$

$$w_e \leftarrow \text{proj}_{w>0}\{w_e - \eta \nabla_e \Omega\}$$

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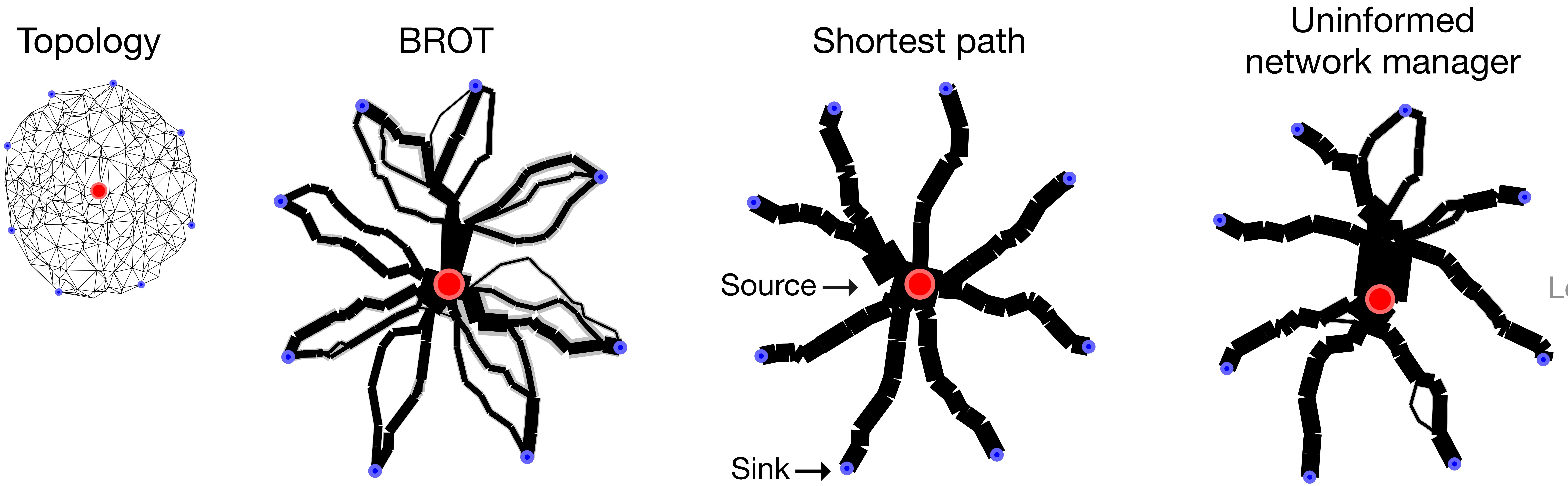
BROT

Results

Contribution 2: BROT successfully trade offs transport cost and traffic mitigation



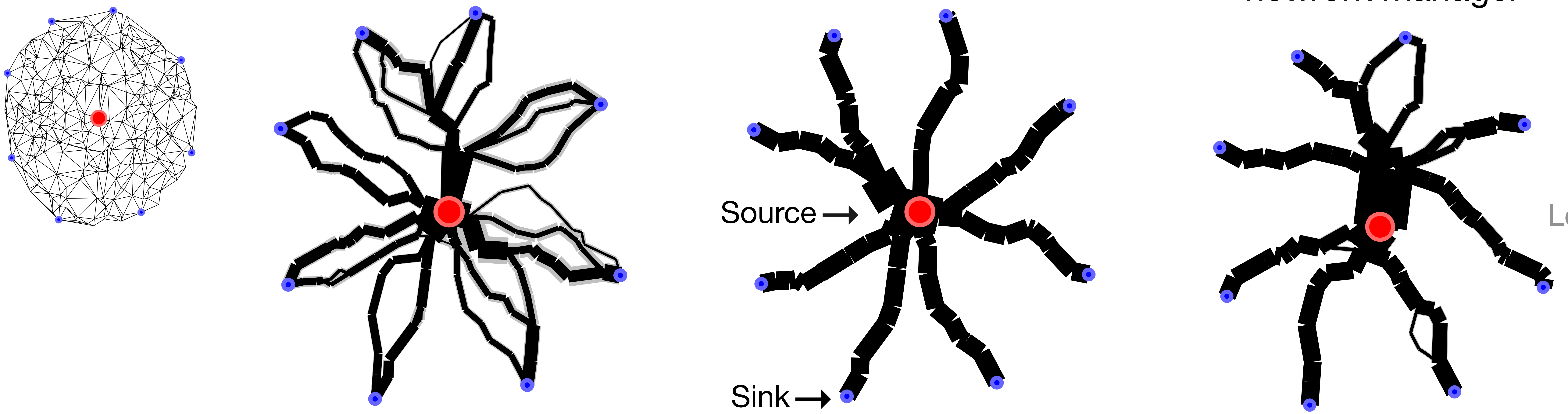
Adapted from Lonardi and De Bacco (2023)

	BROT	Shortest path	Uninformed network manager
Ω	Low	High	Higher
J	Moderate	Low	Low

Results

Contribution 2: BROT successfully trade offs transport cost and traffic mitigation

Topology	BROT	Shortest path	Uninformed network manager



Adapted from Lonardi and De Bacco (2023)

	BROT	Shortest path	Uninformed network manager
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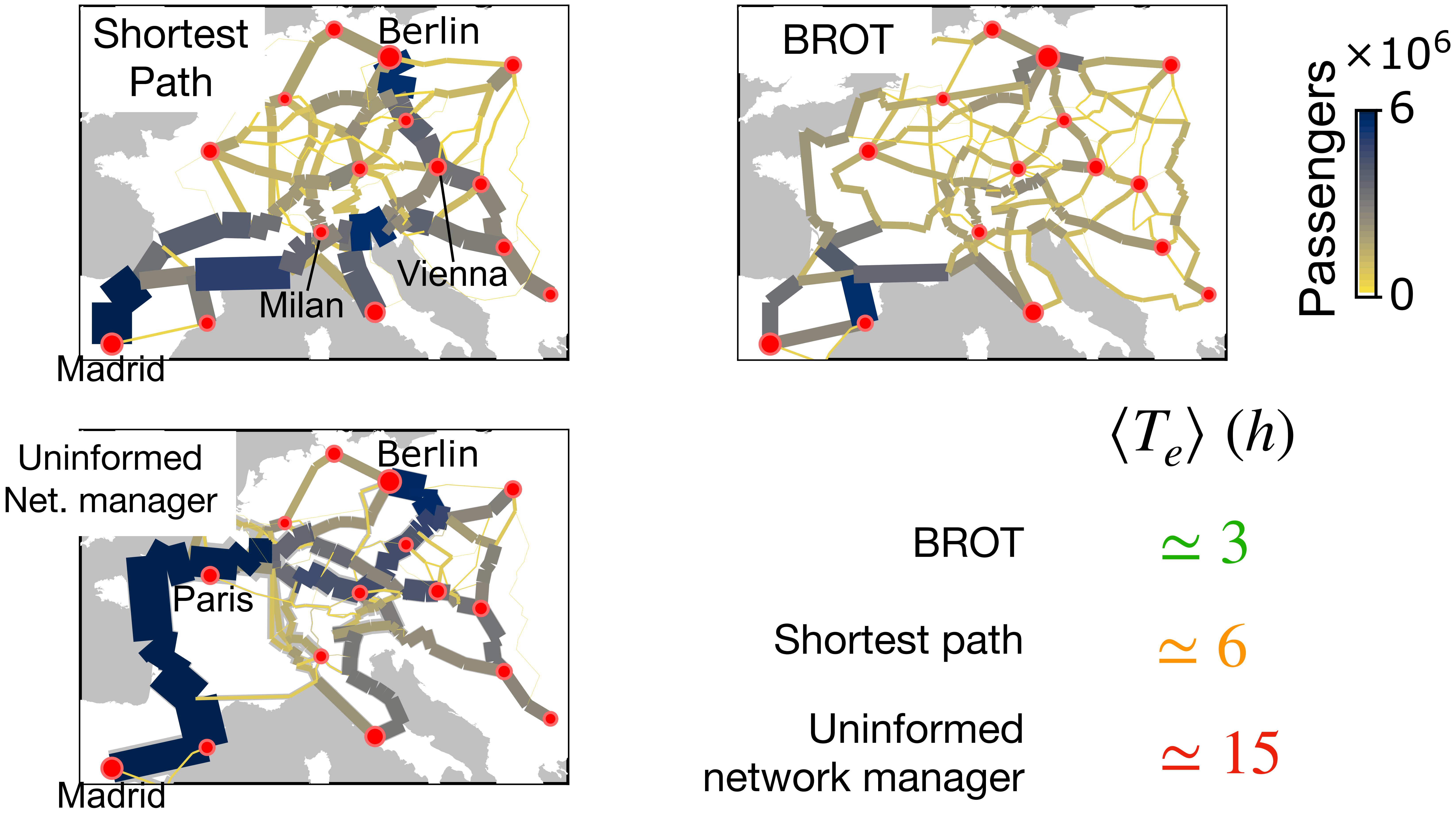
Results

Contribution 3: BROT reduces travel times on International European Highways



	$\langle T_e \rangle$ (h)
BROT	$\simeq 3$
Shortest path	$\simeq 6$
Uninformed network manager	$\simeq 15$

Take aways

Questions

1) Can we find adaptation rules to solve the bilevel optimization problem?

2) Does adaptation shed light on transport network properties?

Answers

Contribution 1: Closed-form adaptation rules

Contribution 2: BROT successfully trade offs transport cost and traffic mitigation

Contribution 3: BROT reduces travel times on International European Highways

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(MPI IS)

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BROT

Thank you!

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🌐 aleable.github.io

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