Max Planck Institute for Intelligent Systems - Physics for Inference and Optimization

Immiscible Color Flows in Optimal Transport Networks for Image Classification

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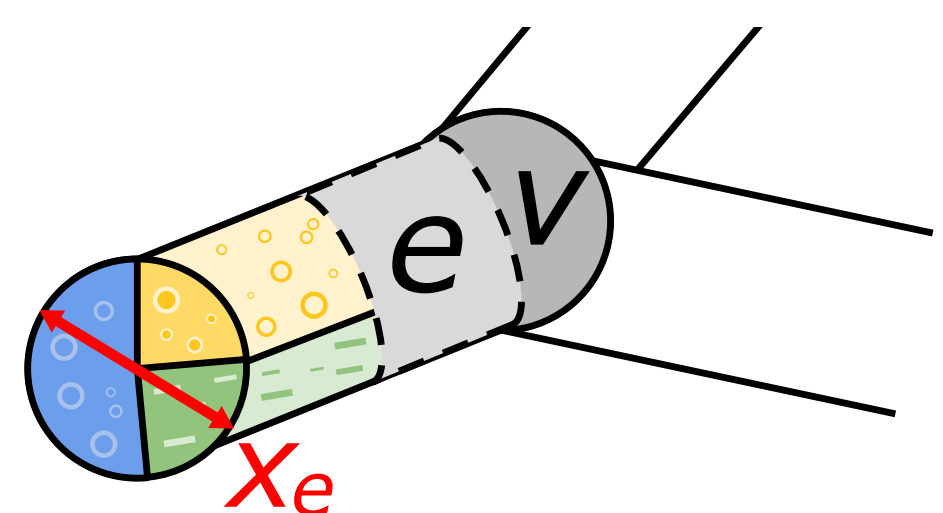
arXiv [6]

[7]

Problem: supervised classification

Goal:

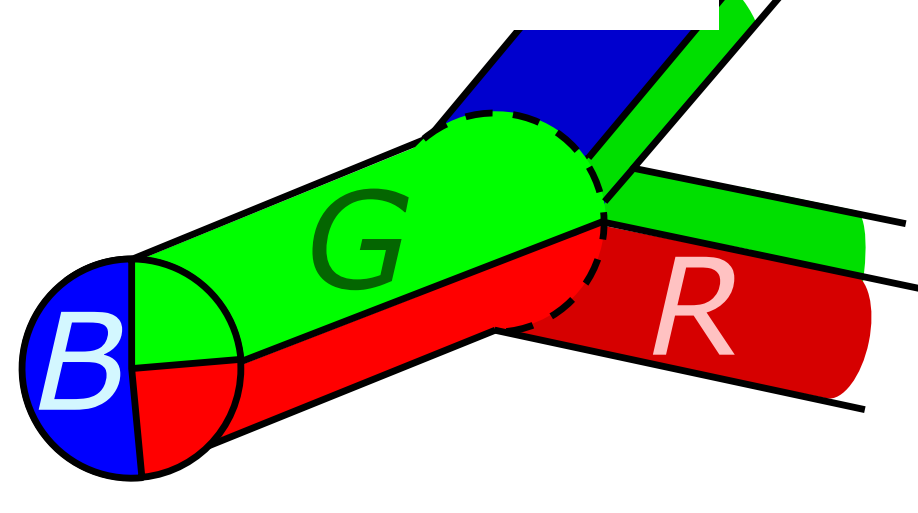→ Use physics principles to boost image classification

Methods:

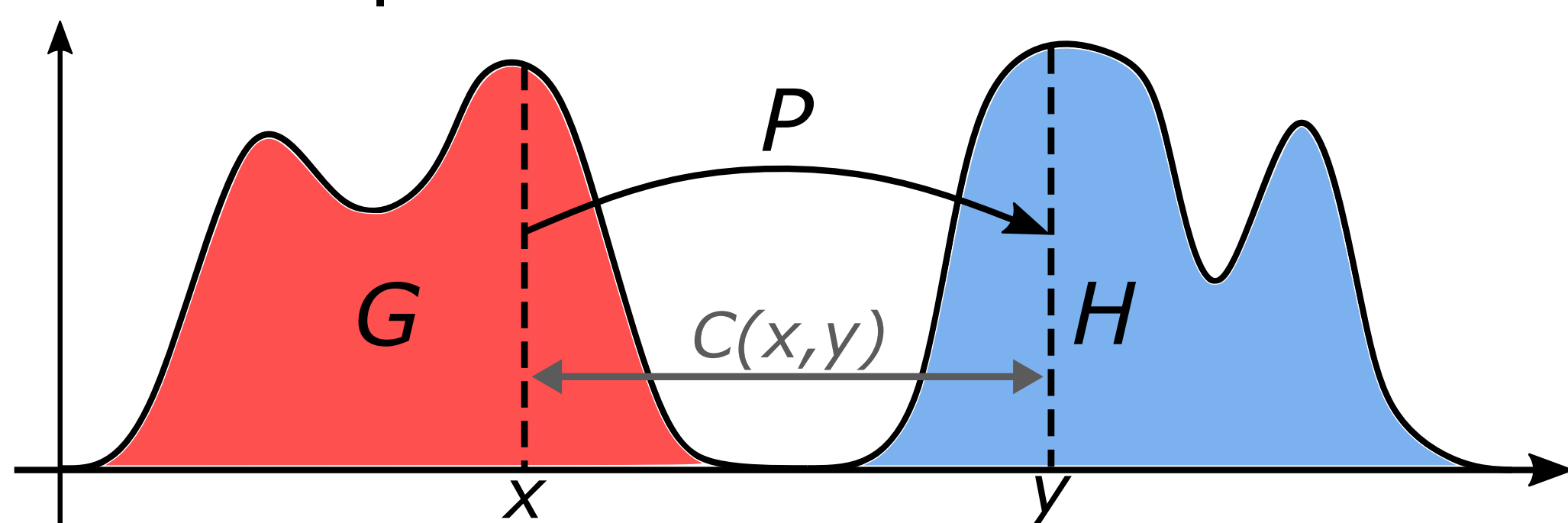⚒ Colors of images are multicommodity flows transported in a network [1]

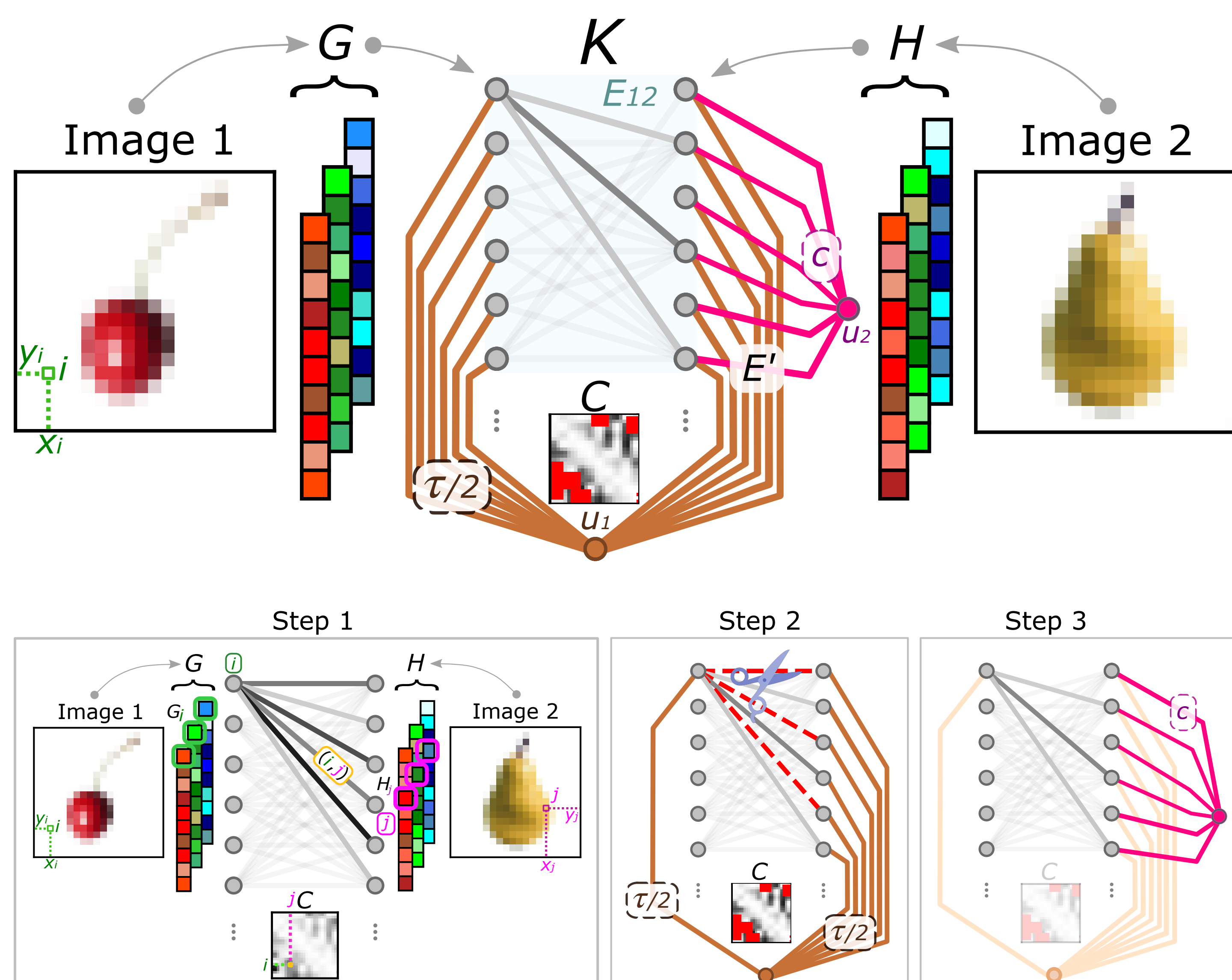immiscible fluids   colors

⚒ Optimal Transport

⚒ Formal setup

Find a transport tensor $P^\star$ solving

$$\min_P \sum_{ij} ||P_{ij}||_2^\Gamma C_{ij}$$

with $0 < \Gamma < 4/3$ regularization parameter

Efficient network construction

1. K, edge cost $C_{ij} = (1-\theta)\,||v_i - v_j||_2 + \theta\,||G_i - H_j||_1$

2. Link cutting, $\min\{C_{ij}, \tau\}$ [2]: $O(mn) \to O(m+n)$

3. Relax mass balance [2]: $\sum_i H_{ia} \neq \sum_j G_{ja}$

Optimal Transport dynamics (MODI)

Color flows are controlled by

$$\sum_{j \in \partial i} L_{ij}[x]\phi_j^a = S_i^a \qquad \forall i \in V, a = 1, \dots, M$$

$$\frac{dx_e}{dt} = x_e^\beta \frac{||\phi_i - \phi_j||_2^2}{C_e^2} - x_e \qquad \forall e = (i,j) \in E$$

with edges' capacities $x_e \geq 0$, net mass $S_i^a = G_{ia} - H_{ia}$, weighted Laplacian $L_{ij}[x] = \sum_e (x_e/C_e)B_{ie}B_{je}$, and B signed incidence matrix of K. Pressure potentials ϕ_i^a act on nodes, potential-based flows are $P_e^a = x_e(\phi_i^a[x] - \phi_j^a[x])/C_e$, and $\Gamma = 2(2-\beta)/(3-\beta)$

Results: classification task

	Algorithm	Class. acc. [%] (↑)	
[3]	MODI RGB (ours)	**62.2**	
	Sinkhorn RGB [5]	58.4	
	Sinkhorn GS [5]	54.3	
	MODI GS (ours)	53.6	
[4]	MODI RGB (ours)	**75.0**	
	Sinkhorn RGB [5]	69.6	
	MODI GS (ours)	64.3	
	Sinkhorn GS [5]	60.7	

The impact of colors
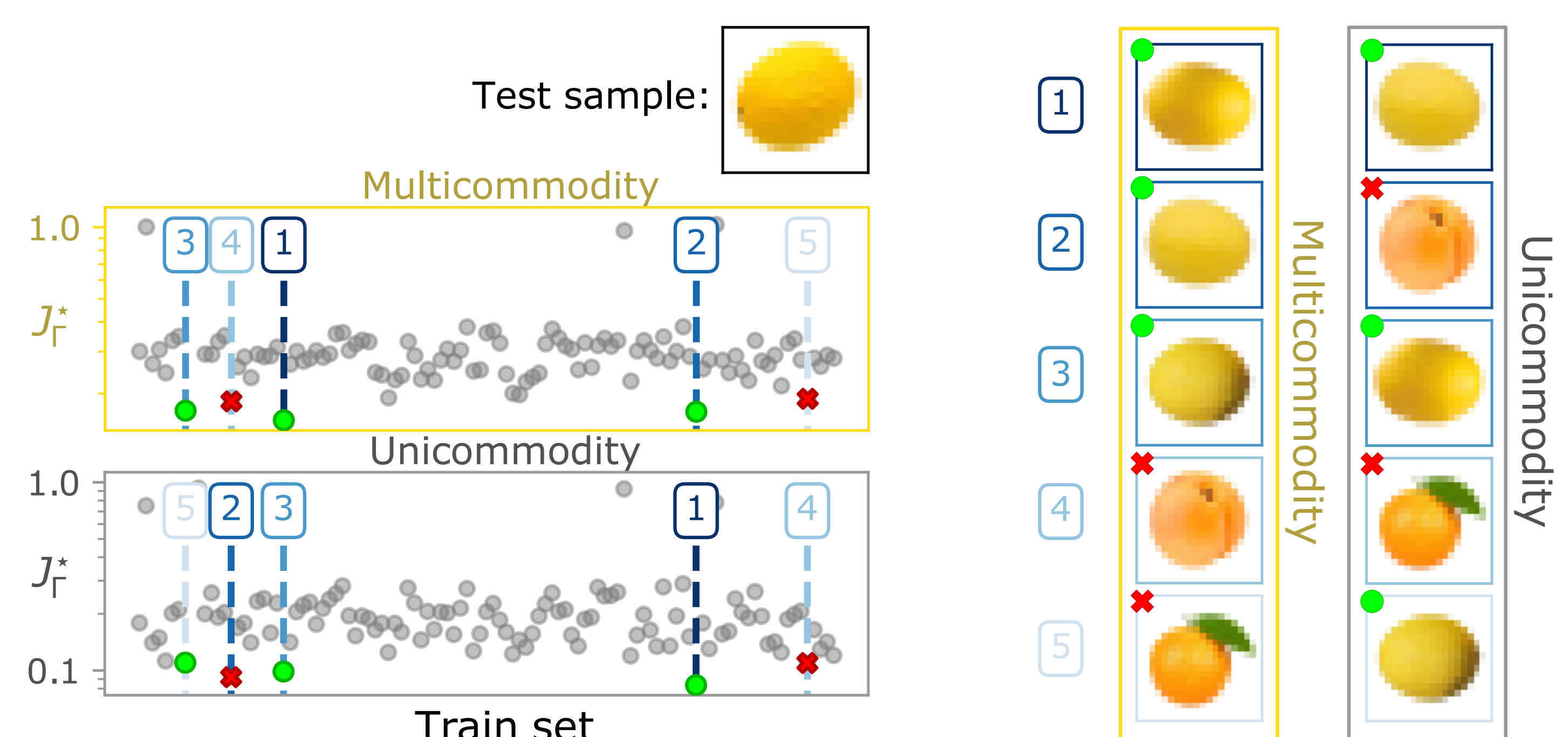
[1] Lonardi, Facca, Putti, and De Bacco, Phys. Rev. Research (2021)

[2] Pele and Werman, ECCV 2008 (2008)

[3] Seeland, Rzanny, Alaqraa, Wäldchen, and Mäder, PloS one (2017)

[4] Macanhã, Eler, Garcia, and Junior, Advances in Intelligent Systems and Computing (2018)

[5] Cuturi, NeurIPS 2013 (2013)

[6] Lonardi, Baptista, and Debacco arXiv:2205.02938 (2022)

[7] MODI: open-source code

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